Angles and Right Triangles

1. Determine the value of the six trig functions given the point on the coordinate plane is (-3, -9)

$$r^2 = x^2 + y^2 \rightarrow r^2 = (-3)^2 + (-9)^2 \rightarrow r = \sqrt{90} = 3\sqrt{10}$$

$$\sin\theta = \frac{y}{r} = \frac{-9}{3\sqrt{10}} = \frac{-3\sqrt{10}}{10}, \cos\theta = \frac{x}{r} = \frac{-3}{3\sqrt{10}} = \frac{-\sqrt{10}}{10}, \tan\theta = \frac{y}{x} = \frac{-9}{-3} = 3$$

$$\csc\theta = \frac{r}{y} = \frac{3\sqrt{10}}{-9} = -\frac{\sqrt{10}}{3}, \sec\theta = \frac{r}{x} = \frac{3\sqrt{10}}{-3} = -\sqrt{10}, \cot\theta = \frac{x}{y} = \frac{-3}{-9} = \frac{1}{3}$$

2. Determine the five remaining trig functions given that $\cos x = -7/12$

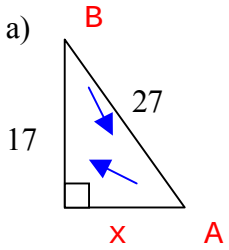
$$r^2 = x^2 + y^2 \rightarrow (12)^2 = (-7)^2 + (y)^2 \rightarrow y = \sqrt{95}$$

$$\sin\theta = \frac{y}{r} = \frac{\sqrt{95}}{12} =, \tan\theta = \frac{y}{x} = \frac{\sqrt{95}}{-7} = -\frac{\sqrt{95}}{7}$$

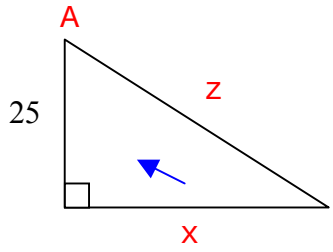
$$\csc\theta = \frac{r}{y} = \frac{12}{\sqrt{95}} = \frac{12\sqrt{95}}{95}, \sec\theta = \frac{r}{x} = \frac{12}{-7} = -\frac{12}{7}, \cot\theta = \frac{x}{y} = \frac{-7}{\sqrt{95}} = -\frac{7\sqrt{95}}{95}$$

3. Determine the missing components of the right triangle: (label the triangle in a manner that best suits your needs).

a)



b)



$$c^2 = a^2 + b^2 \rightarrow 27^2 = 17^2 + x^2 \rightarrow x = 20.97$$

$$\sin A = \frac{opp}{hyp} = \frac{17}{27} = .6296 \rightarrow A = 39.02°$$

$$\cos B = \frac{adj}{hyp} = \frac{17}{27} = .6296 \rightarrow B = 50.98°$$

$$\angle A = 90° - \angle B = 90° - 54° = 36°$$

$$\sin\angle B = \frac{opp}{hyp} \rightarrow \sin 54° = \frac{25}{z} \rightarrow z = \frac{25}{\sin 54°} = 30.9$$

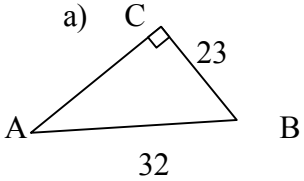
$$\tan\angle B = \frac{opp}{adj} \rightarrow \tan 54° = \frac{25}{x} \rightarrow x = \frac{25}{\tan 54°} = 18.16$$

2. Determine the indicated trig functions from the given triangle.

`

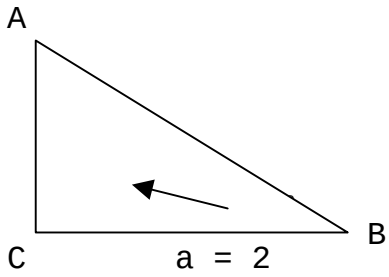
$$c^2 = a^2 + b^2 \rightarrow 32^2 = 23^2 + AC^2 \rightarrow AC = \sqrt{495} = 22.24$$

a)



I) sin ∠A = 23/32
2) cot ∠B = 32/23
3) sec∠A = 32/22.24

3. Determine the missing components of a right triangle given the following information: $\angle C = 90°$, a = 2 and $\angle B = 18°$.

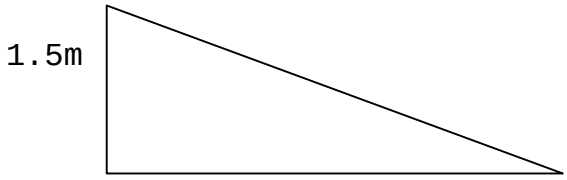


$$\angle A = 90° - \angle B° = 90° - 18° = 72°$$

$$\tan\angle B = \frac{opp}{adj} \rightarrow \tan 18° = \frac{AC}{2} \rightarrow 2\tan 18° = AC \rightarrow AC = 0.65$$

$$\cos\angle B = \frac{adj}{hyp} \rightarrow \cos 18° = \frac{2}{AB} \rightarrow AB = \frac{2}{\cos 18°} \rightarrow AB = 2.1$$

4. If one end of a loading ramp is 1.5 meters from the ground and the other end makes an angle of 9° with the ground, find the length of the ramp.



$$\sin\theta = \frac{opp}{hyp} \rightarrow \sin 9° = \frac{1.5m}{ramp} \rightarrow ramp = \frac{1.5m}{\sin °9} = 9.59m$$

Special Angles:

1. Simplify without using a calculator:

a) $\sin 30° + \tan 60°$

$$\frac{1}{2} + \sqrt{3}$$

b) $2\cos 45 + \sec 60$

$$2\left(\frac{\sqrt{2}}{2}\right) + 2 = \sqrt{2} + 2$$

c) $\sec 120 + \tan 240 - (\csc 315)^2$

d) $(\sin 45 + \cos 30)(\cos 45 - \sin 120)$

$$-\sec 60° + \tan 60° - (-\csc 45°)^2$$

$$-2 + \sqrt{3} - \left(-\frac{\sqrt{2}}{2}\right)^2 = -2 + \sqrt{3} - \frac{1}{2}$$

$$= -\frac{5}{3} + \sqrt{3}$$

$$\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\right) = \frac{2}{4} - \frac{3}{4} = -\frac{1}{4}$$

d) $\sec\frac{\pi}{6} + \tan\frac{2\pi}{3}$

e) $\tan\frac{11\pi}{6} * \sin\frac{5\pi}{4} * \csc\frac{5\pi}{3}$

$$\sec 30° + \tan 120° =$$

$$\sec 30° - \tan 60°$$

$$\frac{2\sqrt{3}}{3} - \sqrt{3} = -\frac{\sqrt{3}}{3}$$

$$\tan 330° * \sin 225° * \csc 300° =$$

$$-\tan 30° * -\sin 45° * -\csc 60° =$$

$$-\frac{\sqrt{3}}{3} * -\frac{\sqrt{2}}{2} * -\frac{2\sqrt{3}}{3} = -\frac{\sqrt{2}}{3}$$

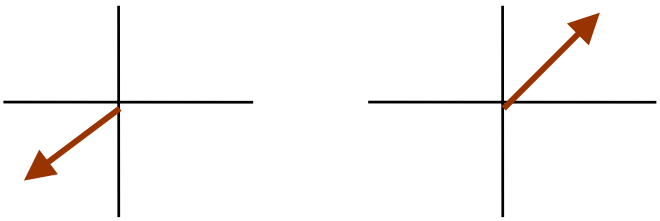
f) $\sec\frac{\pi}{3}\left(\cot\frac{7\pi}{6}+\sin\frac{2\pi}{3}\right)$

g) $\sin\frac{15\pi}{3}+\sec\frac{11\pi}{4}-\csc\frac{13\pi}{3}$

$\sec 60°(\cot 210°+\sin 120°)=$
$\sec 60°(\cot 30°+\sin 60°)=$
$2\left(\frac{\sqrt{3}}{3}+\frac{\sqrt{3}}{2}\right)=2\left(\frac{5\sqrt{3}}{6}\right)=\frac{5\sqrt{3}}{2}$

$\sin 900°+\sec 495°-\csc 780°=$
$\sin 180°+\sec 135°-\csc 60°=$
$\sin 180°-\sec 45°-\csc 60°=$
$0-\sqrt{2}-\frac{2\sqrt{3}}{3}=-\sqrt{2}-\frac{2\sqrt{3}}{3}$

1. Draw an angle in standard position having:
   a) a degree measure of 203° b) a degree measure of -310°



2. Give two positive and two negative coterminal angles for:
   a) 215° 575, 935, -145, -505

   b) -313° –673, –1033, 47, 407

3. Give the reference angle for each of the following:
   a) 160° = 20 b) -156° = 24 c) 203° = 23 d) -805° = 85

4. Convert the following angle into "pi" and radian measures
   156° $\frac{156}{180}\pi, 2.722$

5. Convert the following pi measures into degree and radian measure
   7/9 pi 140, 2.44

6. Convert the following radian measure into degree and pi measure
   5.43 $311.14, \frac{311.14}{180}\pi$

7. Determine the missing information:
   a) the distance the point (4, -2) is from the origin

$r^2=x^2+y^2\rightarrow r^2=4^2+(-2)^2\rightarrow r=\sqrt{20}$

   b) the x coordinate of the point that is 10 units from the origin that has a y coordinate of -4 and exists in the fourth quadrant.

$r^2=x^2+y^2\rightarrow 10^2=x^2+(-4)^2\rightarrow x=\sqrt{86}$

8. Determine the following values:

a) sin 127° = .7986 b) tan 212° = .6248 c) csc 209° = –2.06

9. Convert 69.497 degrees into degrees, minutes and seconds 69°29'49.2"

10. Convert 12 degrees 19 minutes 56 seconds into degrees. 12.3322°

11. If the central angle is 40 degrees and the radius is 8 cm find the arc length.

$$S = r\theta \rightarrow S = 8cm \cdot \frac{40°}{57.3°} = 5.58cm$$

12. If the arc length of a partial rotation of a wheel is 156 cm and the radius of the wheel is 12 cm, find the measure of the central angle in degrees.

$$S = r\theta \rightarrow 156cm = 12cm \cdot \theta \rightarrow \theta = \frac{156cm}{12cm} = 13 \rightarrow \theta = 13 \cdot 57.3° = 744.9°$$

13. How far will a point on a wheel travel if the radius of the wheel is 20 cm and the central angle has a measure of 3368 degrees.

$$S = r\theta \rightarrow S = 20cm \cdot \frac{3368°}{57.3°} = 1175.57cm$$