GEOMETRIC SEQUENCES OR PROGRESSIONS

1. In the following exercises state if a geometric sequence and give the common ratio.

a) 1, 3, 7, 15	no	b) 1, 1, 1, 1	yes, r = 1	c) 4, 2, 1, 1/2	yes, r = 1/2
d) 1, 0, 1, 0	no	e) -2, 2, -2, 2	yes, r = -1	f)3, 1, 1/3, 1/9	yes, r = 1/3
g) 1, -1, 1, -1	yes, r = -1	h) 1, 2, 4, 8	yes, r = 2	i) a, abc, ab^2c^2	yes, r = bc
j) 1, 0, 0, 0	no	k) 2, 4, 8, 16	yes, r = 2	l) a - b, a, a + b	no
m) 8, -2, -1/8	no	n) 1/9, 1/3, 1, 3	yes, r = 3	o) 1/2, 1/4, 1/6, 1/8	no

2. State the first four terms of the given geometric sequence.

1. a = 1, r = 2 $\underline{1},1\cdot 2=\underline{2},2\cdot 2=\underline{4},4\cdot 2=\underline{8}$

2. a = 27/4, r = 2/3 $\underline{27/4},\frac{27}{4}\cdot\frac{2}{3}=\underline{9/2},\frac{9}{2}\cdot\frac{2}{3}=\underline{3},3\cdot\frac{2}{3}=\underline{2}$

3. a = 1/2, r = -3 $\underline{1/2},\frac{1}{2}\cdot -3=\underline{-3/2},\frac{-3}{2}\cdot -3=\underline{9/2},\frac{9}{2}\cdot -3=\underline{-27/2}$

4. a = 8/25, r = 5/2 $\underline{8/25},\frac{8}{25}\cdot\frac{5}{2}=\underline{4/5},\frac{4}{5}\cdot\frac{5}{2}=\underline{2},2\cdot\frac{5}{2}=\underline{5}$

3. Find the nth term of the indicated geometric sequence.

1. $a=3,r=-2,n=5:l=ar^{n-1}\Rightarrow l=3\cdot(-2)^{5-1}\Rightarrow l=3\cdot(-2)^4\Rightarrow l=3\cdot 16=48$
2. $a=2,r=\frac{-1}{9},n=6:l=ar^{n-1}\Rightarrow l=2\cdot(-\frac{1}{9})^{6-1}\Rightarrow l=2\cdot(-\frac{1}{9})^5\Rightarrow l=2\cdot-\frac{1}{59049}=-\frac{2}{59049}$
3. $a=15,r=-1,n=21:l=ar^{n-1}\Rightarrow l=15\cdot(-1)^{21-1}\Rightarrow l=15\cdot(-1)^{20}\Rightarrow l=15\cdot 1=15$
4. $a=-3,r=\frac{1}{4},n=5:l=ar^{n-1}\Rightarrow l=-3\cdot\left(\frac{1}{4}\right)^{5-1}\Rightarrow l=-3\cdot\left(\frac{1}{4}\right)^4\Rightarrow l=-3\cdot\frac{1}{256}=-\frac{3}{256}$

4. Find the specified term in the given sequence.

1. 8th, in 1/16, -1/8, 1/4...
   $a=\frac{1}{16},r=-2,n=8$
   $l=ar^{n-1}\Rightarrow l=\frac{1}{16}(-2)^{8-1}\Rightarrow l=\frac{1}{16}(-2)^7\Rightarrow l=\frac{1}{16}\cdot(-128)=-8$
2. 9th, in 20, -2, 0.2,…
   $a=20,r=-\frac{1}{10},n=9$
   $l=ar^{n-1}\Rightarrow l=20\left(-\frac{1}{10}\right)^{9-1}\Rightarrow l=20\left(-\frac{1}{10}\right)^8\Rightarrow l=20\cdot\left(\frac{1}{10^8}\right)=\frac{20}{10^8}=2\times 10^{-7}$
3. 6th, in 1, 4, 16,...
   $a=1,n=6,r=4$
   $l=ar^{n-1}\Rightarrow l=1(4)^{6-1}\Rightarrow l=1(4)^5\Rightarrow l=1\cdot(1024)=1024$
4. 7th, in 0.0003, 0.03, 3,...

$a = .0003, r = 100, n = 7$

$l = ar^{n-1} \Rightarrow l = .0003(100)^{7-1} \Rightarrow l = .0003(100)^6 \Rightarrow l = 3\times10^{-4}\cdot10^{12} = 3\times10^8$

5. Insert the indicated number of geometric means between the given numbers.

1. two, between 1 and 125

$a = 1, l = 125, n = 4$

$l = ar^{n-1} \Rightarrow 125 = 1(r)^{4-1} \Rightarrow 125 = 1(r)^3 \Rightarrow \sqrt[3]{125} = \sqrt[3]{r^3} \Rightarrow r = 5$

$1, 1\cdot5 = \underline{5}, 5\cdot5 = \underline{25}, 125$

2. three, between 3/25 and 25/27

$a = \frac{3}{25}, n = 5, l = \frac{25}{27}$

$l = ar^{n-1} \Rightarrow \frac{25}{27} = \frac{3}{25}(r)^{5-1} \Rightarrow \frac{25}{27} = \frac{3}{25}(r)^4 \Rightarrow \frac{625}{81} = r^4$

$\Rightarrow \sqrt[4]{\frac{625}{81}} = \sqrt[4]{r^4} \Rightarrow r = \pm\frac{5}{3}$

$\frac{3}{25}, \frac{3}{25}\cdot\pm\frac{5}{3} = \underline{\pm\frac{1}{5}}, \pm\frac{1}{5}\cdot\pm\frac{5}{3} = \underline{\frac{1}{3}}, \frac{1}{3}\cdot\pm\frac{5}{3} = \underline{\pm\frac{5}{9}}, \frac{25}{27}$

3. two, between -2/9 and -18

$a = -\frac{2}{9}, n = 4, l = -18$

$l = ar^{n-1} \Rightarrow -18 = -\frac{2}{9}(r)^{4-1} \Rightarrow -18 = -\frac{2}{9}(r)^3 \Rightarrow 81 = r^3 \Rightarrow \sqrt[3]{81} = \sqrt[3]{r} \Rightarrow r = 3\sqrt[3]{3}$

$-\frac{2}{9}, -\frac{2}{9}\cdot3\sqrt[3]{3} = \underline{\frac{-2\sqrt[3]{3}}{3}}, \frac{-2\sqrt[3]{3}}{3}\cdot3\sqrt[3]{3} = \underline{-2\sqrt[3]{3^2}}, -18$

4. three, between -5/8 and -2/125

$a = -\frac{5}{8}, n = 5, l = -\frac{2}{125}$

$l = ar^{n-1} \Rightarrow -\frac{2}{125} = -\frac{5}{8}(r)^{5-1} \Rightarrow -\frac{2}{125} = -\frac{5}{8}(r)^4 \Rightarrow \frac{16}{625} = r^4 \Rightarrow$

$\sqrt[4]{\frac{16}{625}} = \sqrt[4]{r} \Rightarrow r = \pm\frac{2}{5}$

$-\frac{5}{8}, -\frac{5}{8}\cdot-\frac{2}{5} = \underline{\frac{1}{4}}, \frac{1}{4}\cdot-\frac{2}{5} = \underline{-\frac{1}{10}}, -\frac{1}{10}\cdot-\frac{2}{5} = \underline{\frac{1}{25}}, -\frac{2}{125}$

6. Answer the following:

1. Which term in the geometric progression 1/24, -1/6, 2/3,... is 512/3?

$a = \frac{1}{24}, r = -4, l = \frac{512}{3}$

$l = ar^{n-1} \Rightarrow \frac{512}{3} = \frac{1}{24}\cdot(-4)^{n-1} \Rightarrow 4096 = (-4)^{n-1} \Rightarrow (-4)^6 = (-4)^{n-1} \Rightarrow 6 = n-1 \Rightarrow n = 7$

2. The first term of a geometric sequence is 162, the common ratio is 1/3 and there is $a_n = 2/9$. What is the value of n?

$a = 162, r = \frac{1}{3}, l = \frac{2}{9}$

$l = ar^{n-1} \Rightarrow \frac{2}{9} = 162\cdot\left(\frac{1}{3}\right)^{n-1} \Rightarrow \frac{1}{729} = \left(\frac{1}{3}\right)^{n-1} \Rightarrow (\frac{1}{3})^6 = \left(\frac{1}{3}\right)^{n-1} \Rightarrow 6 = n-1 \Rightarrow n = 7$

3.The fifth term of a geometric sequence is 3/4 and the common ratio is 3/2. What is the first term?

$r = 3/2, l = 3/4, n = 5$

$l = ar^{n-1} \Rightarrow 3/4 = a \cdot (3/2)^{5-1} \Rightarrow 3/4 = a \cdot (3/2)^4 \Rightarrow 3/4 = a \cdot (81/16) \Rightarrow 3/4 \cdot 16/81 = a \Rightarrow a = 4/27$

4.The seventh term of a geometric sequence is 1875 and the fifth term is 75. What is the first term?

Mini sequence

$5th = 75, 7th = 1875, n = 3$

$l = ar^{n-1} \Rightarrow 1875 = 75r^{3-1} \Rightarrow 25 = r^2 \Rightarrow r = \pm 5$

$l = 1875, n = 7, r = \pm 5$

$l = ar^{n-1} \Rightarrow 1875 = a \cdot (\pm 5)^{7-1} \Rightarrow 1875 = a \cdot (\pm 5)^6 \Rightarrow 1875 = a \cdot (15625) \Rightarrow a = 1875/15625 = 3/25$

5. If the fourth term of a geometric sequence is 5/3 and the seventh term is -625/81, what is the third term?

Mini sequence

$4th = 5/3, 7th = -625/81, n = 4$

$l = ar^{n-1} \Rightarrow -625/81 = 5/3 r^{4-1} \Rightarrow -125/27 = r^3 \Rightarrow r = -5/3$

$l = 7th = -625/81, n = 5, r = -5/3$

$l = ar^{n-1} \Rightarrow -625/81 = a \cdot (-5/3)^{5-1} \Rightarrow -625/81 = a \cdot (-5/3)^4 \Rightarrow -625/81 = a \cdot (625/81) \Rightarrow a == -1$

or if we multiply the 3rd term by the ratio it must equal the 4th term. Let 3rd term = x

$x \bullet -5/3 = 5/3 \Rightarrow x = -1$

7. Answer the following:

1. Find the first term of the geometric progression whose 6th and 7th terms are 32/9 and 64/27 respectively.

$r = \dfrac{64/27}{32/9} = \dfrac{64 \cdot 9}{27 \cdot 32} = \dfrac{2}{3}$

$l = ar^{n-1} \Rightarrow 64/27 = a(2/3)^{7-1} \Rightarrow 64/27 = a(2/3)^6 \Rightarrow 64/27 = a(64/729) \Rightarrow$

$64/27 \cdot 729/64 = a \Rightarrow a = 27$

2. 2m - 8, 2m + 4 5m -2 are successive terms of a geometric progression. Find the value of m.

Geometric mean, Use the following formula: $\sqrt{a \cdot l} = g.m.$

$\sqrt{(2m-8)(5m-2)} = 2m+4 \Rightarrow \left(\sqrt{(2m-8)(5m-2)}\right)^2 = (2m+4)^2 \Rightarrow$

$10m^2 - 44m + 16 = 4m^2 + 16m + 16 \Rightarrow$

$6m^2 - 60m = 0 \Rightarrow 6m(m-10) = 0 \Rightarrow m = 0, m = 10$