Probability

1. A jar contains 4 blue, 6 green, 5 red and 1 yellow marble. If one marble is drawn at random from the jar, find the probability that the marble is:

   a) blue $B = \frac{4}{16}$ b) not blue $\overline{B} = \frac{12}{16}$ c) red $R = \frac{5}{16}$ d) not red $\overline{R} = \frac{11}{16}$

   e) green $G = \frac{6}{16}$ f) red or green $R + G = \frac{5}{16} + \frac{6}{16}$ g) yellow $Y = \frac{1}{16}$

   h) blue or yellow $B + Y = \frac{4}{16} + \frac{1}{16}$

2. A letter is chosen at random from the word BANANA. Find the probability that the letter is:

   a) an A $P(A) = \frac{3}{6}$ b) a consonant $P(cons) = \frac{3}{6}$ c) an N $P(N) = \frac{2}{6}$

   d) not a B $P(\overline{B}) = \frac{5}{6}$ e) an A or an N $P(A) + P(N) = \frac{3}{6} + \frac{2}{6}$

3. If all of the letters of the ABOUT are arranged at random in a line, find the probability that the arrangement will:

   a) spell the word ABOUT $\frac{1 \cdot 1 \cdot 1 \cdot 1 \cdot 1}{5 \cdot 4 \cdot \cdot 3 \cdot 2 \cdot 1}$

   b) have the A at either end and the O in the middle

   A at beginning and O in middle + O in middle and A at end = $\frac{1 \cdot 3 \cdot 1 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} + \frac{3 \cdot 2 \cdot 1 \cdot 1 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

   c) not spell the word ABOUT

   use the complement = 1 - P(letters spell about) = 1 - $\frac{1 \cdot 1 \cdot 1 \cdot 1 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

   d) have three vowels

   $$\underline{VVV}CC + C\underline{VVV}C + CC\underline{VVV} = \frac{3 \cdot 2 \cdot 1 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} + \frac{2 \cdot 3 \cdot 2 \cdot 1 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} + \frac{2 \cdot 1 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

   together $or \underline{VVV}CC \cdot 3 = \frac{3 \cdot 2 \cdot 1 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \cdot 3$

   because the 3 vowels could occupy three different groupings as above

   e) start and end with a vowel $V\_\_\_V = \frac{3 \cdot 3 \cdot 2 \cdot 1 \cdot 2}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

   f) have two consonants side by side $CCVVV + VCCVV + VVCCV + VVVCC \Rightarrow CCVVV \cdot 4 = \frac{2 \cdot 1 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \cdot 4$

   g) start and end with a consonant $C\_\_\_C = \frac{2 \cdot 3 \cdot 2 \cdot 1 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

   h) start with AB $AB\_\_\_ = \frac{1 \cdot 1 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

4. A carton of one dozen eggs contains three that are rotten. If a set of two eggs is chosen at random from the carton, find the probability of selecting:

   a) two rotten eggs $RR = \frac{3}{12} \cdot \frac{2}{11}$

b) 1 rotten egg and 1 good egg $RG + GR = \frac{3}{12}\cdot\frac{9}{11} + \frac{9}{12}\cdot\frac{3}{11}$

c) two good eggs $GG = \frac{9}{12}\cdot\frac{8}{11}$

5. A jar contains 6 blue, 5 green and 8 yellow marbles. If a set of three marbles is chosen at random from the jar, find the probability that your selection contains:

a) 3 blue $BBB = \frac{6}{19}\cdot\frac{5}{18}\cdot\frac{4}{17}$

b) 3 marbles, not all blue $complement = 1 - P(BBB) = 1 - \frac{6}{19}\cdot\frac{5}{18}\cdot\frac{4}{17}$

c) 3 green $GGG = \frac{5}{19}\cdot\frac{4}{18}\cdot\frac{3}{17}$

d) 3 marbles, not all green $complement = 1 - P(GGG) = 1 - \frac{5}{19}\cdot\frac{4}{18}\cdot\frac{3}{17}$

e) 3 yellow $YYY = \frac{8}{19}\cdot\frac{7}{18}\cdot\frac{6}{17}$

f) 1 of each color

$BGY = \frac{6}{19}\cdot\frac{5}{18}\cdot\frac{8}{17}\cdot 6$ because these three colors can be arranged $3\cdot 2\cdot 1$ ways

g) no blue no blue means any other color (G or Y) $\Rightarrow \frac{13}{19}\cdot\frac{12}{18}\cdot\frac{11}{17}$

h) at least one blue $complement = 1 - P(\text{no blue}) = 1 - \frac{13}{19}\cdot\frac{12}{18}\cdot\frac{11}{17}$

i) no green no green means any other color (B or Y) $= \frac{14}{19}\cdot\frac{13}{18}\cdot\frac{12}{17}$

j) no yellow no yellow means any other color (B or G) $= \frac{11}{19}\cdot\frac{10}{18}\cdot\frac{9}{17}$

k) at least 2 green

$GG\_ + G\_G + \_GG + GGG = \frac{5}{19}\cdot\frac{4}{18}\cdot\frac{14}{17} + \frac{5}{19}\cdot\frac{14}{18}\cdot\frac{4}{17} + \frac{14}{19}\cdot\frac{5}{18}\cdot\frac{4}{17} + \frac{5}{19}\cdot\frac{4}{18}\cdot\frac{3}{17}$

6. A 5-card hand is dealt from a standard deck of 52 cards. Find the probability that a 5-card hand contains:

a) only black cards $\frac{26\cdot 25\cdot 24\cdot 23\cdot 22}{52\cdot 51\cdot 50\cdot 49\cdot 48}$

b) 4 aces

$AAAA\_ + AAA\_A + AA\_AA + A\_AAA + \_AAAA \Rightarrow$ possible arrangements

$\frac{4\cdot 3\cdot 2\cdot 1\cdot 48}{52\cdot 51\cdot 50\cdot 49\cdot 48}\cdot 5$

c) no aces $\frac{48\cdot 47\cdot 46\cdot 45\cdot 44}{52\cdot 51\cdot 50\cdot 49\cdot 48}$

d) 4 cards of the same value

$\left[AAAA\_ + AAA\_A + AA\_AA + A\_AAA + \_AAAA\right] \Rightarrow$

possible arrangements $\cdot$ 13 different kinds

$$\frac{4\cdot3\cdot2\cdot1\cdot48}{52\cdot51\cdot50\cdot49\cdot48}\cdot5\cdot15$$

e) no spades $\frac{39\cdot38\cdot37\cdot36\cdot35}{52\cdot51\cdot50\cdot49\cdot48}$

f) 3 clubs and 2 diamonds

3 different clubs and 2 different diamonds results in $5\cdot4\cdot3\cdot2\cdot1$ possible arrangements

$$\frac{13\cdot12\cdot11\cdot13\cdot12}{52\cdot51\cdot50\cdot49\cdot48}\cdot120$$

g) all diamonds $\frac{13\cdot12\cdot11\cdot10\cdot9}{52\cdot51\cdot\cdot50\cdot49\cdot48}$

g) 5 cards, all of the same suit $\overset{\text{4 different suits}}{\frac{13\cdot12\cdot11\cdot10\cdot9}{52\cdot51\cdot\cdot50\cdot49\cdot48}\cdot4}$

i) all face cards 12 face cards $\Rightarrow \frac{12\cdot11\cdot10\cdot9\cdot8}{52\cdot51\cdot50\cdot49\cdot48}$

j) 3 aces and 2 sevens

3 different clubs and 2 different diamonds results in $5\cdot4\cdot3\cdot2\cdot1$ possible arrangements

$$\frac{4\cdot3\cdot2\cdot4\cdot3}{52\cdot51\cdot50\cdot49\cdot48}\cdot120$$

7. If a coin is tossed 3 times find the probability of tossing:

a) exactly one head or exactly two heads

$HTT + THT + TTH + HHT + HTH + THH \Rightarrow 6$ arrangements

$$\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot6\cdot$$

b) exactly one head and exactly one tail 0 because there are three tosses

c) at least one head or at least one tail

at least one head $\Rightarrow HTT + THT + TTH + HHT + HTH + THH + HHH \Rightarrow 7\,possiblilities$

$$\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot7$$

at least one tail $\Rightarrow THH + HTH + HHT + TTH + THT + HTT + TTT \Rightarrow 7\,possiblilities$

$$\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot7$$

note there are 6 arrangements found in each argument, therefore we have interection $\therefore$

$$\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot7+\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot7-\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot6$$

d) at least two heads or no tails

at least two heads $\Rightarrow HHT + HTH + THH + HHH \Rightarrow$ 4 arrangements $\Rightarrow \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot 4$

no tails $\Rightarrow HHH \Rightarrow \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}$

intersection 3 heads $\Rightarrow HHH \Rightarrow \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}$

$P(\text{at least two heads}) + P(\text{no tails}) = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot 4 + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} - \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}$

e) a head on the first toss or a tail on the last toss

a head on the first toss $\Rightarrow HTT + HTH + HHT + HHH \Rightarrow$ 4 arrangements $\Rightarrow \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot 4$

a tail on the last toss $\Rightarrow HTT + THT + HHT + TTT \Rightarrow$ 4 arrangements $\Rightarrow \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot 4$

intersection $\Rightarrow HTT + HHT \Rightarrow$ 2 arrangements $\Rightarrow \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot 2$

$P(\text{a head on the first toss}) + P(\text{a tail on the last toss}) = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot 4 + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot 4 - \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot 2$

8. A set of 2 cards is chosen from a standard deck of 52 cards. Find the probability that both cards are:

a) black or red $BB + RR = \frac{26}{52}\cdot\frac{25}{51} + \frac{26}{52}\cdot\frac{25}{51}$

Intersection because 6 face cards are black $\frac{6}{52}\cdot\frac{5}{51}$

b) black or face card

$BB + FF = \frac{26}{52}\cdot\frac{25}{51} + \frac{12}{52}\cdot\frac{11}{51} - \frac{6}{52}\cdot\frac{5}{51}$

c) black or hearts $BB + HH = \frac{26}{52}\cdot\frac{25}{51} + \frac{13}{52}\cdot\frac{12}{51}$

intersection 2 aces that are black $\Rightarrow \frac{2}{51}\cdot\frac{1}{51}$

d) black or aces

$BB + AA = \frac{26}{51}\cdot\frac{25}{51} + \frac{4}{51}\cdot\frac{3}{51} - \frac{2}{51}\cdot\frac{1}{51}$

9. Marbles numbered from 1 to 15 are placed in a bag. If a set of three marbles is drawn from a bag, find the probability that:

a) all three marbles are odd or all three show numbers greater than 6

intersection odd numbers greater than 6 $\{7,9,11,13,15\} \Rightarrow \frac{5}{15}\cdot\frac{4}{14}\cdot\frac{3}{13}$

all numbers odd $\Rightarrow \frac{8}{15}\cdot\frac{7}{14}\cdot\frac{6}{13}$

numbers greater than 6 $\Rightarrow \frac{9}{15}\cdot\frac{8}{14}\cdot\frac{7}{13}$

$P(\text{all numbers odd}) + P(\text{numbers greater than 6}) = \frac{8}{15}\cdot\frac{7}{14}\cdot\frac{6}{13} + \frac{9}{15}\cdot\frac{8}{14}\cdot\frac{7}{13} - \frac{5}{15}\cdot\frac{4}{14}\cdot\frac{3}{13}$

b) all three marbles show numbers greater than 7 or all marbles show numbers less than 2

greater than 7 $=\frac{8}{15}\cdot\frac{7}{14}\cdot\frac{6}{13}$

less than 2 = 0 (only 1 mable less than 2)

P(greater than)+P(less than 2)=$\frac{8}{15}\cdot\frac{7}{14}\cdot\frac{6}{13}$

10. A coin is tossed and a die is rolled. Find the probability of obtaining:

a) a tail and a 5 $T5=\frac{1}{2}\cdot\frac{1}{6}$ b) a head and a number greater than 5 $H(>5)=\frac{1}{2}\cdot\frac{1}{6}$

c) a head and an even number $HEv=\frac{1}{2}\cdot\frac{3}{6}$ d) a tail and a number less than 1 $T(<1)=\frac{1}{2}\cdot 0$

11. A contains 3 red, 5 yellow, 2 green and 6 blue marbles. If one marble is chosen at random and replaced, then a second marble is chosen at random, find the probability of obtaining:

a) 2 red marbles $RR=\frac{3}{16}\cdot\frac{3}{16}$

b) 2 marbles of different colors

because of the number of possibilities (RY, YR, RG, GR, RB, BR...) it is better to consider the ways to get both the same color (RR, YY, GG, BB) and then use the complement $\Rightarrow 1-\left(\frac{3}{16}\cdot\frac{2}{15}+\frac{5}{16}\cdot\frac{4}{15}+\frac{2}{16}\cdot\frac{1}{15}+\frac{6}{16}\cdot\frac{5}{15}\right)$

c) 2 blue marbles $BB=\frac{6}{16}\cdot\frac{5}{15}$

d) a red and a yellow in either order $RW+WR=\frac{3}{16}\cdot\frac{5}{15}+\frac{5}{16}\cdot\frac{3}{15}$

e) 2 marbles of the same color $RR+YY+GG+BB=\left(\frac{3}{16}\cdot\frac{2}{15}+\frac{5}{16}\cdot\frac{4}{15}+\frac{2}{16}\cdot\frac{1}{15}+\frac{6}{16}\cdot\frac{5}{15}\right)$

f) f) a blue and a green marble in that order $BG=\frac{6}{16}\cdot\frac{2}{15}$

12. Suppose that the probability that you will pass a math test is 9/10, pass a chem test is 3/7 and pass a social test is 4/5. If these events are independent, find the probability that you:

a) pass all three $MCS=\frac{9}{10}\cdot\frac{3}{7}\cdot\frac{4}{5}$

b) pass only one $M\overline{C}\overline{S}+\overline{M}C\overline{S}+\overline{M}\overline{C}S=\frac{9}{10}\cdot\frac{4}{7}\cdot\frac{1}{5}+\frac{1}{10}\cdot\frac{3}{7}\cdot\frac{1}{5}+\frac{1}{10}\cdot\frac{4}{7}\cdot\frac{4}{5}$

c) pass at least two

$MC\overline{S}+M\overline{C}S+\overline{M}CS+MCS=\frac{9}{10}\cdot\frac{3}{7}\cdot\frac{1}{5}+\frac{9}{10}\cdot\frac{4}{7}\cdot\frac{4}{5}+\frac{1}{10}\cdot\frac{3}{7}\cdot\frac{4}{5}+\frac{9}{10}\cdot\frac{3}{7}\cdot\frac{4}{5}$

d) fail all $\overline{M}\overline{C}\overline{S}=\frac{1}{10}\cdot\frac{4}{7}\cdot\frac{1}{5}$

13. If two dice are rolled, find the probability of rolling:

a) a sum of 6, given that doubles were rolled $\frac{1}{36}$

b) a sum of 10, given that doubles were not rolled $\frac{2}{36}$

14. Two cards are drawn from a deck without replacement. Find the probability that:

a) both are face cards $FF = \frac{12}{52} \cdot \frac{11}{51}$

b) both are aces $AA = \frac{4}{52} \cdot \frac{3}{51}$

c) two diamonds are drawn $DD = \frac{13}{52} \cdot \frac{12}{51}$

d) a king and a queen are drawn in any order $KQ + QK = \frac{4}{52} \cdot \frac{3}{51} + \frac{4}{52} \cdot \frac{3}{51}$

e) the second card is black, given that the first card is a club $C(Black) = \frac{13}{52} \cdot \frac{25}{51}$

f) the second card is red, given that the first card is a heart $H(red) = \frac{13}{52} \cdot \frac{25}{51}$

g) the second card is a face card, given that the first card is a queen $QF = \frac{4}{52} \cdot \frac{11}{51}$

h) the second card is a three, given the first card is not a three $\bar{3}(3) = \frac{48}{52} \cdot \frac{4}{51}$