Probability

1. A box contains 3 baseballs, 7 softballs and 11 tennis balls.
   i) What is the probability that a ball selected at random will be:
      a) a tennis ball?, $T = \frac{11}{21}$ b) a baseball?, $B = \frac{3}{21}$ c) a softball? $S = \frac{7}{21}$
   ii) If two balls are selected at random , what would be the probability that:
      a) the first is a softball and the second a tennis ball? $ST = \frac{7}{21} \cdot \frac{11}{20}$
      b) the first is a softball and the second is a baseball? $SB = \frac{7}{21} \cdot \frac{3}{20}$
      c) that both balls are tennis balls? $TT = \frac{11}{21} \cdot \frac{10}{20}$
      d) that a softball and baseball are drawn? $SB + BS = \frac{7}{21} \cdot \frac{3}{20} + \frac{3}{21} \cdot \frac{7}{20}$

2. Two cards are drawn at random from a standard deck of 52 cards. What is the probability that:
   a) both are hearts? $HH = \frac{13}{52} \cdot \frac{12}{51}$
   b) both are tens? $10 \circ 10 = \frac{4}{52} \cdot \frac{3}{51}$
   c) both are black? $BB = \frac{26}{52} \cdot \frac{25}{51}$
   d) the first card is a two and the second a five? $2 \circ 5 = \frac{4}{52} \cdot \frac{4}{51}$
   e) the first is a prime number and the second a face card?
   Prime $= 2,3,5,7$ and Face $= J,Q,K \Rightarrow PF = \frac{16}{52} \cdot \frac{12}{51}$
   f) the first card is between two and six and the second is between seven and nine inclusively?\
   between 2 and 6 $= 3, 4, 5$ and between 7 and 9 $= 8 \Rightarrow 2-6 \circ 7-9 = \frac{12}{52} \cdot \frac{4}{51}$

3. A collection of 15 transistors contains 3 that are defective. If 2 transistors are selected at random, what is the probability that:
   a) both are good? $GG = \frac{12}{15} \cdot \frac{11}{14}$
   b) both are defective? $DD = \frac{3}{15} \cdot \frac{2}{14}$
   c) that at least one is defective?

$$GD + DG + DD = \frac{12}{15}\cdot\frac{3}{14} + \frac{3}{15}\cdot\frac{12}{14} + \frac{3}{15}\cdot\frac{2}{14}$$

$$\text{or } 1 - GG = 1 - \frac{12}{15}\cdot\frac{11}{15} \text{ (complement of the event)}$$

4. A number is picked at random from the integers 1 through 50. Find the probability of each of the following:

   a) an odd integer? $P(O) = \frac{25}{50}$

   b) an integer divisible by 5? $P(5) = \frac{10}{50}$

   c) a multiple of four? $P(4) = \frac{12}{50}$

   d) a perfect square? $P(PS) = \frac{7}{50}$

   e) a prime number? $P(prime) = \frac{11}{50}$

   f) a number between 1 and 50, inclusively? $P(1-50, incl) = \frac{50}{50}$

   g) the number 60 $P(60) = \frac{0}{50}$

5. A new phone is being installed. Find the probability that the final three digits in the telephone number will be even?

   $$P(\text{final 3 digits even}) = \frac{5}{10}\cdot\frac{5}{10}\cdot\frac{5}{10} \text{ (with replacement)}$$

   $$P(\text{final 3 digits even}) = \frac{5}{10}\cdot\frac{4}{10}\cdot\frac{3}{10} \text{ (without replacement)}$$

6. Fifty tickets, numbered consecutively from 1 to 50 are placed in a box. What is the probability that in 4 separate drawings, the following selections will occur?

   a) 4 odd numbers, if replacement occurs $P(4 \text{ odd numbers}) = \frac{25}{50}\cdot\frac{25}{50}\cdot\frac{25}{50}\cdot\frac{25}{50}$

   b) 4 odd numbers, if no replacement occurs $P(4 \text{ odd numbers}) = \frac{25}{50}\cdot\frac{24}{50}\cdot\frac{23}{50}\cdot\frac{22}{50}$

   c) 4 prime number $P(4 \text{ prime numbers}) = \frac{11}{50}\cdot\frac{10}{50}\cdot\frac{9}{50}\cdot\frac{8}{50}$

   d) 4 even numbers $P(4 \text{ even numbers}) = \frac{25}{50}\cdot\frac{24}{50}\cdot\frac{23}{50}\cdot\frac{22}{50}$