Probability

1. a) Write out the sample space when flipping one coin. H, T
   b) What is the probability of flipping a head? $\frac{1}{2}$
   c) What is the probability of flipping a tail? $\frac{1}{2}$

2. a) Write out the sample space when flipping two coins. HH, HT, TH, TT

   b) What is the probability of flipping two heads? $H \Rightarrow H = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$
   c) What is the probability of flipping a head and then a tail in that order?
   $H \Rightarrow T = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$
   d) What is the probability of flipping a head and a tail?
   $H \Rightarrow T = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} + T \Rightarrow H = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \triangleright \frac{2}{4}$
   e) What is the probability of flipping two tails? $T \Rightarrow T = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

3. a) Write out the sample space when flipping three coins. HHH, HHT, HTH,THH, HTT, THT, TTH, TTT
   b) What is the probability of flipping three heads? $H \Rightarrow H \Rightarrow H = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \triangleright \frac{1}{8}$
   c) What is the probability of flipping two heads and then a tail?
   $H \Rightarrow H \Rightarrow T = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \triangleright \frac{1}{8}$
   d) What is the probability of flipping a head, then a tail and then a head?
   $H \Rightarrow T \Rightarrow H = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \triangleright \frac{1}{8}$
   e) What is the probability of flipping two heads and a tail?
   $H \Rightarrow H \Rightarrow T = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + H \Rightarrow T \Rightarrow H = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + T \Rightarrow H \Rightarrow H = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \triangleright \frac{3}{8}$

   f) What is the probability of flipping anything but three tails?
   $T \Rightarrow T \Rightarrow T = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \triangleright \frac{1}{8}$

4. a) Write out the sample space when rolling one die. 1, 2, 3, 4, 5, 6
   b) What is the probability of rolling a three? $\frac{1}{6}$
   c) What is the probability of rolling a five? $\frac{1}{6}$
   d) What is the probability of rolling an even number? $\frac{3}{6} = \frac{1}{2}$

e) What is the probability of rolling an odd number? $3/6 = 1/2$

f) What is the probability of rolling a prime number?
Prime Numbers $= 2,3,5 \Rightarrow 3/6 = 1/2$

5. a) Write out the sample space when rolling two dice? 2 – 1, 3 - 2, 4 - 3, 5 - 4, 6 - 5, 7 - 6, 8 - 5, 9 - 4, 10 – 3, 11 - 2, 12 - 1

b) What is the probability of rolling a ten? $3/36 = 1/12$

c) What is the probability of rolling a six? $5/36$

d) What is the probability of rolling a prime number?
Prime Numbers $= 2,3,5,7,11 \Rightarrow 15/36 = 5/12$

e) What is the probability of rolling a number greater than seven?
Greater than seven $= 8,9,10,11,12 = 15/36$

f) What is the probability of rolling a number greater than 4 but less than or equal to 10? $4 < x \le 10 = 27/36 = 3/4$

g) What is the probability of rolling a number that is even? $2,4,6,8,10,12 = 18/36 = 1/2$

6. A box contains 6 red, 4 green and 4 blacks marbles. What is the probability of drawing the following: total outcomes = 14, no replacement

a) a red marble?, $6/14 = 3/7$ a green marble? $4/14 = 2/7$, a black marble? $4/14 = 2/7$, a white marble? $0/14 = 0$, a marble that is either red, green or black? $14/14 = 1$

b) a red then a green marble? $R \Rightarrow G = 6/14 \cdot 4/13 = 12/91$

c) a red and a green marble?
$R \Rightarrow G = 6/14 \cdot 4/13 + G \Rightarrow R = 4/14 \cdot 6/13 \triangleright 12/91 + 12/91 = 24/91$

d) a red, green and black marble in that order (with replacement)? Without replacement?

with replacement $R \Rightarrow G \Rightarrow B = \frac{6}{14} \cdot \frac{4}{14} \cdot \frac{4}{14}$

without replacement $R \Rightarrow G \Rightarrow B = \frac{6}{14} \cdot \frac{4}{13} \cdot \frac{4}{12}$

e) Write out the ways in which a red, a green and a black marble may be drawn from the box replacement?, without replacement?
same for both $RGB, RBG, BRG, BGR, GRB, GBR$

f) of drawing a red, a green and a black in any order (with replacement)?, without replacement?

replacement $RGB, RBG, BRG, BGR, GRB, GBR$

$$\frac{6}{14}\cdot\frac{4}{14}\cdot\frac{4}{14}+\frac{6}{14}\cdot\frac{4}{14}\cdot\frac{4}{14}+\frac{4}{14}\cdot\frac{6}{14}\cdot\frac{4}{14}+\frac{4}{14}\cdot\frac{4}{14}\cdot\frac{6}{14}+\frac{4}{14}\cdot\frac{6}{14}\cdot\frac{4}{14}+\frac{4}{14}\cdot\frac{4}{14}\cdot\frac{6}{14}$$

replacement $RGB, RBG, BRG, BGR, GRB, GBR$

$$\frac{6}{14}\cdot\frac{4}{13}\cdot\frac{4}{12}+\frac{6}{14}\cdot\frac{4}{13}\cdot\frac{4}{12}+\frac{4}{14}\cdot\frac{6}{13}\cdot\frac{4}{12}+\frac{4}{14}\cdot\frac{4}{13}\cdot\frac{6}{12}+\frac{4}{14}\cdot\frac{6}{13}\cdot\frac{4}{12}+\frac{4}{14}\cdot\frac{4}{13}\cdot\frac{6}{12}$$

g) of drawing three red marbles? $RRR=\frac{6}{14}\cdot\frac{5}{13}\cdot\frac{4}{12}$

h) of drawing 2 green marbles? $GG=\frac{4}{14}\cdot\frac{3}{13}$

i) of drawing 4 black marbles? $BBBB=\frac{4}{14}\cdot\frac{3}{13}\cdot\frac{2}{12}\cdot\frac{1}{11}$

j) of drawing 3 white marbles? $WWW=0$

7. Using a regular bridge deck (52 cards), determine the probability of the following:

a) drawing a heart?, $H=\frac{13}{52}=\frac{1}{4}$ a diamond? $D=\frac{13}{52}=\frac{1}{4}$

b) drawing a black card? $B=\frac{26}{52}=\frac{1}{2}$

c) drawing a queen?, $Q=\frac{4}{52}=\frac{1}{13}$ ten? $10=\frac{4}{52}=\frac{1}{13}$ an ace?, $A=\frac{4}{52}=\frac{1}{13}$

a seven? $7=\frac{4}{52}=\frac{1}{13}$

d) drawing a red card on the first card and a black card on the second?

$RB=\frac{26}{52}\cdot\frac{26}{51}=\frac{13}{51}$

e) drawing a queen on the first card and a ten on the second card?

$Q10=\frac{4}{52}\cdot\frac{4}{51}=\frac{1}{13}\cdot\frac{4}{51}$

f) drawing a face card on the first card and a two on the second card?

$F2=\frac{12}{52}\cdot\frac{4}{51}=\frac{3}{13}\cdot\frac{4}{51}=\frac{1}{13}\cdot\frac{4}{17}$

g) drawing a two, three or five on the first card? $2 \text{ or } 3 \text{ or } 5=\frac{4}{52}+\frac{4}{52}+\frac{4}{52}=\frac{12}{52}=\frac{3}{13}$

f) drawing a king or queen on the first card and a four or seven on the second card? $KG=\frac{4}{52}+\frac{4}{52}=\frac{8}{52}$ and $47=\frac{4}{52}+\frac{4}{52}=\frac{8}{52}\Rightarrow\frac{8}{52}\cdot\frac{8}{52}=\frac{2}{13}\cdot\frac{2}{13}=\frac{4}{169}$

8. If the probability of Norman passing Math 30 is 4/7, of Bill passing Math 30 is 3/5 and of Joan passing math 30 is 7/9, find the probability that:

a) that all three will pass? $NBJ = \frac{4}{7}\cdot\frac{3}{5}\cdot\frac{7}{9}$

b) Norman will pass but Bill and Joan will not? $N\overline{B}\overline{J} = \frac{4}{7}\cdot\frac{2}{5}\cdot\frac{2}{9}$

c) Joan will pass but Norman and Bill will not? $\overline{N}\overline{B}J = \frac{3}{7}\cdot\frac{2}{5}\cdot\frac{7}{9}$

b) that no one will pass? $\overline{N}\overline{B}\overline{J} = \frac{3}{7}\cdot\frac{2}{5}\cdot\frac{2}{9}$

c) Write out the possibilities of only one passing. $N\overline{B}\overline{J}$ or $\overline{N}B\overline{J}$ or $\overline{N}\overline{B}J$

d) that only one will pass?

$N\overline{B}\overline{J}$ or $\overline{N}B\overline{J}$ or $\overline{N}\overline{B}J = \frac{4}{7}\cdot\frac{2}{5}\cdot\frac{2}{9} + \frac{3}{7}\cdot\frac{3}{5}\cdot\frac{2}{9} + \frac{3}{7}\cdot\frac{2}{5}\cdot\frac{7}{9} \Rightarrow \frac{16}{315} + \frac{18}{315} + \frac{42}{315}$

e) that two out of the three will pass?

$NB\overline{J}$ or $N\overline{B}J$ or $\overline{N}BJ = \frac{4}{7}\cdot\frac{3}{5}\cdot\frac{2}{9} + \frac{4}{7}\cdot\frac{2}{5}\cdot\frac{7}{9} + \frac{3}{7}\cdot\frac{3}{5}\cdot\frac{7}{9} = \frac{24}{315} + \frac{56}{315} + \frac{63}{315}$

9. If the probability that the snow will melt today is 3/7,

a) what is the probability that it will not melt today? $\overline{S} = \frac{4}{7}$

b) what are the odds of the snow melting? $\frac{S}{\overline{S}} = \frac{\frac{3}{7}}{\frac{4}{7}} = \frac{3}{4}$

c) what are the odds of the snow not melting? $\frac{\overline{S}}{S} = \frac{\frac{4}{7}}{\frac{3}{7}} = \frac{4}{3}$

10. If the odds of being successful in this class are 6/11:

a) What is the probability of being successful?

since the odds are $6/11$ of beginning successful, the ratio must be:

$\frac{S}{\overline{S}} = \frac{6}{11} = \frac{\frac{6}{17}}{\frac{11}{17}} \Rightarrow$ the denominator is 17 - the sum of the num and den of the odds

$S = \frac{6}{17}$

b) What is the probability of not being successful? from part (a) $\overline{S} = \frac{11}{17}$