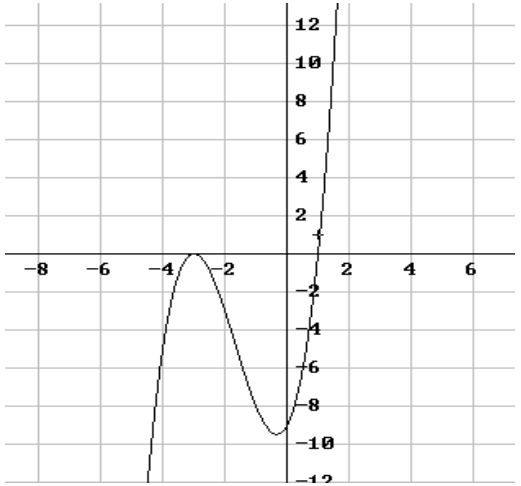
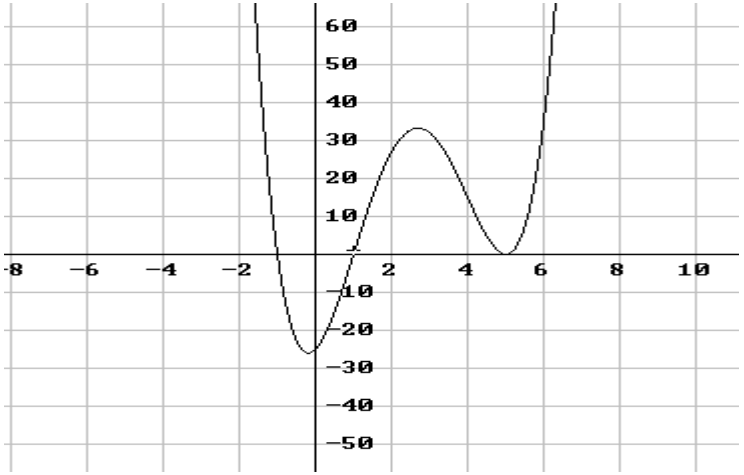
*Polynomial Function Exam*

| *Equation* | $x^3 + 5x^2 + 3x - 9$ | $x^4 - 10x^3 + 24x^2 + 10x - 25$ | | $x^3 + 5x^2 + 3x - 9$ | $x^4 - 10x^3 + 24x^2 + 10x - 25$ |
|---|---|---|---|---|---|
| 1. *degree of equation* | 3 | 4 | *7. possible number of positive real roots* | 1 | 2 |
| 2. *the value of the constant* | -9 | -25 | *8. possible number of negative real roots* | 1 | 1 |
| 3. *the value of the leading coefficient* | 1 | 1 | *9. possible number of imaginary roots* | 0 | 0 |
| 4. *the value of the y intercept* | -9 | -25 | *10. write out the possible factors* | $(x+3)^2(x-1)$ | $(x-1)(x+1)(x-5)^2$ |
| 5. *where the graphs starts* | Lower left | Upper left | *11. identify critical zeros (values, x-intercepts)* | -3, 1 | -1, 1, 5 |
| 6. *where the graph finishes* | Upper right | Upper Right | *12. multiplicity of each factor* | 2 – (x + 3), 1- (x – 1) | 2 – (x - 5), 1- (x – 1), 1 – (x + 1) |

Sketch each graph



1. *Determine the roots of each quadratic equation by the indicated method.*

*a) Factoring*

$2x^2 + 8x - 10 = 0$

$2(x^2 + 4x - 5) = 0$

$2(x+5)(x-1) = 0$

$x + 5 = 0 \text{ or } x - 1 = 0$

$x = -5 \text{ or } x = 1$

$\{-5, 1\}$

*b) Complete the Trinomial Square*

$2x^2 + 3x - 1 = 0$

$x^2 + \frac{3}{2}x = \frac{1}{2}$

$x^2 + \frac{3}{2}x + \frac{9}{16} = \frac{1}{2} + \frac{9}{16} = \frac{1*8+9}{16}$

$\left(x + \frac{3}{4}\right)^2 = \frac{17}{16}$

$\sqrt{\left(x + \frac{3}{4}\right)^2} = \pm\sqrt{\frac{17}{16}}$

$x + \frac{3}{4} = \frac{\pm\sqrt{17}}{4}$

$x = -\frac{3}{4} \pm \frac{\sqrt{17}}{4} \Rightarrow \left\{\frac{-3 \pm \sqrt{17}}{4}\right\}$

*c) Quadratic Formula*

$-4x^2 + 5x + 3 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(5) \pm \sqrt{(5)^2 - 4(-4)(3)}}{2(-4)}$

$x = \frac{-5 \pm \sqrt{25 + 48}}{-8}$

$x = \frac{-5 \pm \sqrt{73}}{-8} \Rightarrow \left\{\frac{-5 \pm \sqrt{73}}{-8}\right\}$

2. *Determine the sum and the product of the roots for the following equation:*

$4x^2 - 5x + 6 = 0$

$$sum = -\frac{b}{a} = -\frac{-5}{4} = \frac{5}{4}, \qquad product = \frac{c}{a} = \frac{6}{4} = \frac{3}{2}$$

3. *Determine the equation of quadratic function if the roots are* $(2 \pm \sqrt{3})$

$x^2 - (r_1 + r_2)x + r_1 r_2 = 0$

$x^2 - (2 + \sqrt{3} + 2 - \sqrt{3}) + (2 + \sqrt{3})(2 - \sqrt{3}) = 0$

$x^2 - (4)x + (4 - 3) = 0$

$x^2 - 4x + 1 = 0$

4. *Determine the value of the discriminant and the nature of the roots of the quadratic function* $5x^2 - 3x - 2 = 0$.

$b^2 - 4ac$

$(-3)^2 - 4(5)(-2)$

$9 + 40$

$49$

→ nature of roots = 2 real rational roots

5. *Determine the roots of one of the following polynomial functions (Do one of the following)*

a) $3x^4 - 8x^2 - 3 = 0$

*Let* $z = x^2$ *by factoring*

$3z^2 - 8z - 3 = 0$

$(3z+1)(z-3) = 0$

$3z+1 = 0 \text{ or } z-3 = 0$

$z = -\frac{1}{3} \text{ or } z = 3$

$\therefore x^2 = -\frac{1}{3} \text{ or } x^2 = 3$

$x = \{\} \text{ or } x = \pm\sqrt{3}$

*Solution Set* $= \{\pm\sqrt{3}\}$

*Let* $z = x^2$ *by quadratic formula*

$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$z = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(-3)}}{2(3)}$

$z = \frac{8 \pm \sqrt{64+36}}{6} = \frac{8 \pm \sqrt{100}}{6} = \frac{8 \pm 10}{6}$

$z = \frac{8+10}{6} = \frac{18}{6} = 3, z = \frac{8-10}{6} = \frac{-2}{6} = -\frac{1}{3}$

*see above for rest of solution*

b) $4x - 4\sqrt{x} - 15 = 0$

*Let* $z = \sqrt{x}$ *by factoring*

$4z^2 - 4z - 15 = 0$

$(2z-5)(2z+3) = 0$

$2z-5 = 0 \text{ or } 2z+3 = 0$

$z = \frac{5}{2} \text{ or } z = -\frac{3}{2}$

$\therefore \sqrt{x} = \frac{5}{2} \text{ or } \sqrt{x} = -\frac{3}{2}$

$x = \frac{25}{4} \text{ or } x = \frac{9}{4}$

*Solution Set* $= \left\{\frac{25}{4}, \frac{9}{4}\right\}$

*Let* $z = \sqrt{x}$ *by quadratic formula*

$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$z = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(-15)}}{2(4)}$

$z = \frac{4 \pm \sqrt{16+240}}{8} = \frac{4 \pm \sqrt{256}}{8} = \frac{4 \pm 16}{8}$

$z = \frac{4+16}{8} = \frac{20}{8} = \frac{5}{2} \text{ and } z = \frac{4-16}{8} = \frac{-12}{8} = -\frac{3}{2}$

*see above for rest of solution*

6. *State the inverse for each of the following*:

a) $5x - 2y = 11$

Inverse $5x - 2y = 11$

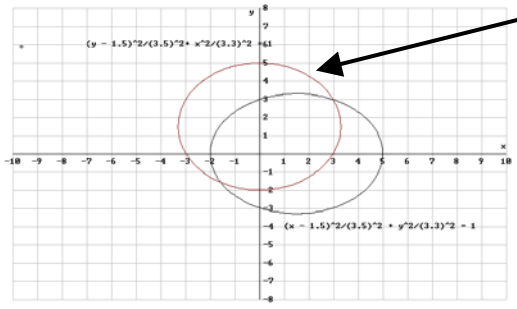
b)

| x | -3 | 1 | 5 |
|---|---|---|---|
| y | 1 | 2 | -4 |

Inverse

| x | 1 | 2 | -4 |
|---|---|---|---|
| y | -3 | 1 | 5 |

c)



7.State the reciprocal for each of the following:

a) $y = 3x - 5$

reciprocal $y = \dfrac{1}{3x-5}$

b)

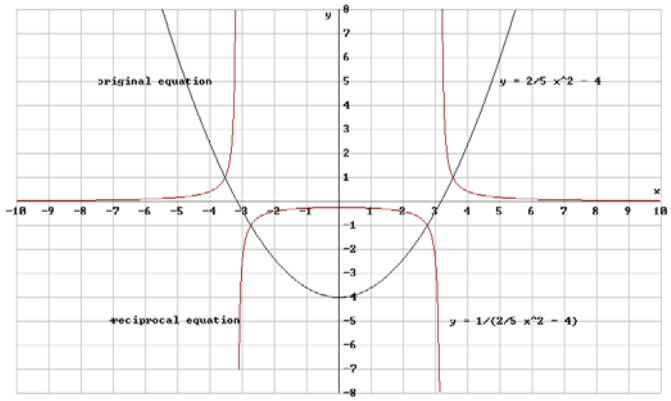
| x | 2 | 4 | 5 |
|---|---|---|---|
| y | 0 | 2 | 7 |

reciprocal

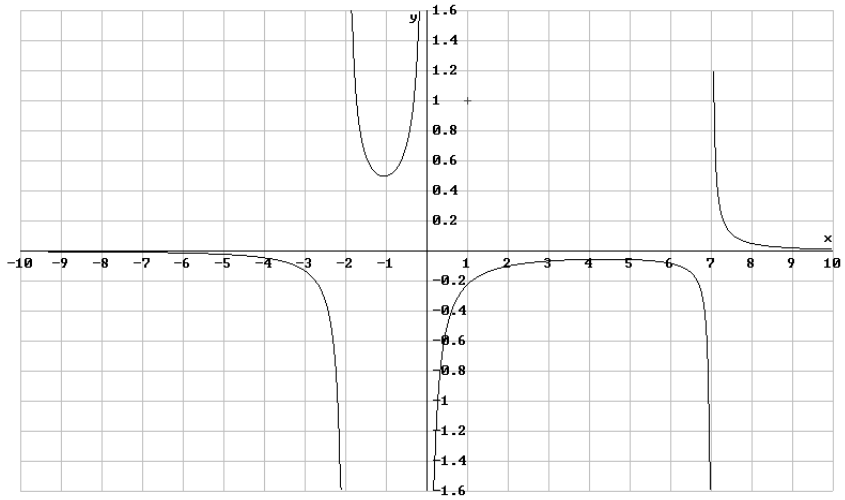
| x | -2 | 4 | 5 |
|---|---|---|---|
| y | 1/0 | ½ | 1/7 |

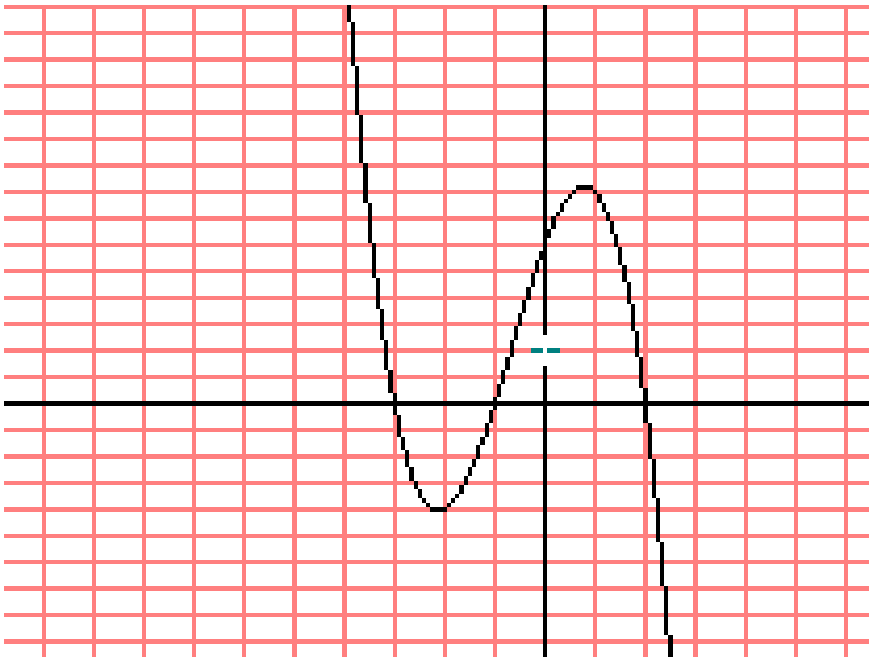
undefined

c)




8. Graph the following rational expression: $y = \dfrac{4}{x^3 - 5x^2 - 14x}$






*Complete the table using the above graphs*

| | | | | | |
|---|---|---|---|---|---|
| *1. possible degree of the function* | 3 | 5 | *7. number of peaks* | 1 | 1 |
| *2. value of the y-intercept* | 6 | -2 | *8. number of valleys* | 1 | 1 |
| *3. number of positive real roots* | 1 | 1 | *9. critical zeros* | -3, -1, 1 | -2, -1, 1 |
| *4. number of negative real roots* | 2 | 2 | *10. the factors containing critical zeros* | (x + 3) (x + 1) (x - 1) | (x + 2) (x +1) (x – 1) |
| *5. number of imaginary roots* | 0 | 0 | *11. the equation* | $y = (x + 3)(x + 1)(x - 1)$ | $y=(x+2)(x+1)^3(x-1$ |
| *6. number of times graph changes direction* | 2 | 2 | *12. multiplicity of each factor* | 1 | 3 - (x + 1) 1 - (x + 2), 1- (x -1 ) |