Matrices

1. Given : $A=\begin{bmatrix}-3 & 5\\ 4 & 2\end{bmatrix}, B=\begin{bmatrix}8 & 3\\ -2 & -4\end{bmatrix}, C=\begin{bmatrix}-5 & -7\\ 2 & 2\end{bmatrix}$

a) A + B – C
b) 3A + 2B
c) C^T
d) A*B + C
e) B^{-1}
f) ∂ A

a) $\begin{bmatrix}-3 & 5\\ 4 & 2\end{bmatrix}+\begin{bmatrix}8 & 3\\ -2 & -4\end{bmatrix}-\begin{bmatrix}-5 & -7\\ 2 & 2\end{bmatrix}=\begin{bmatrix}-3 & 5\\ 4 & 2\end{bmatrix}+\begin{bmatrix}8 & 3\\ -2 & -4\end{bmatrix}+\begin{bmatrix}5 & 7\\ -2 & -2\end{bmatrix}=$

$\begin{bmatrix}(-3)+8+5 & 5+3+7\\ 4+(-2)+(-2) & 2+(-4)+(-2)\end{bmatrix}=\begin{bmatrix}10 & 15\\ 0 & -4\end{bmatrix}$

b) $3\begin{bmatrix}-3 & 5\\ 4 & 2\end{bmatrix}+2\begin{bmatrix}8 & 3\\ -2 & -4\end{bmatrix}=\begin{bmatrix}-9 & 15\\ 12 & 6\end{bmatrix}+\begin{bmatrix}16 & 6\\ -4 & -8\end{bmatrix}=\begin{bmatrix}-9+16 & 15+6\\ 12+(-4) & 6+(-8)\end{bmatrix}=\begin{bmatrix}7 & 21\\ 8 & -2\end{bmatrix}$

c) $\begin{bmatrix}-5 & -7\\ 2 & 2\end{bmatrix}^T=\begin{bmatrix}-5 & 2\\ -7 & 2\end{bmatrix}$

d) $\begin{bmatrix}-3 & 5\\ 4 & 2\end{bmatrix}\cdot\begin{bmatrix}8 & 3\\ -2 & -4\end{bmatrix}+\begin{bmatrix}-5 & -7\\ 2 & 2\end{bmatrix}=\begin{bmatrix}(-3)(8)+(5)(-2) & (-3)(3)+(5)(-4)\\ (4)(8)+(2)(-2) & (4)(3)+(2)(-4)\end{bmatrix}+\begin{bmatrix}-5 & -7\\ 2 & 2\end{bmatrix}=$

$\begin{bmatrix}-34 & -29\\ 28 & 4\end{bmatrix}+\begin{bmatrix}-5 & -7\\ 2 & 2\end{bmatrix}=\begin{bmatrix}-39 & -36\\ 30 & 6\end{bmatrix}$

e) $\begin{bmatrix}8 & 3\\ -2 & -4\end{bmatrix}^{-1}=\frac{1}{(8)(-4)-(-2)(3)}\begin{bmatrix}-4 & -3\\ 2 & 8\end{bmatrix}=\frac{1}{-26}\begin{bmatrix}-4 & -3\\ 2 & 8\end{bmatrix}=\begin{bmatrix}-4/-26 & -3/-26\\ 2/-26 & 8/-26\end{bmatrix}$

f) $(-3)(2)-(5)(4)=-26$

2. Multiply

a) $\begin{bmatrix}4 & 1 & -4\\ -2 & -7 & 3\end{bmatrix}\begin{bmatrix}8 & 5\\ -2 & -1\\ 3 & 6\end{bmatrix}=\begin{bmatrix}(4)(8)+(1)(-2)+(-4)(3) & (4)(5)+(1)(-1)+(-4)(6)\\ (-2)(8)+(-7)(-2)+(3)(3) & (-2)(5)+(-7)(-1)+(3)(6)\end{bmatrix}=$

$\begin{bmatrix}32-2-12 & 20-1-24\\ -16+14+9 & -10+8+18\end{bmatrix}=\begin{bmatrix}18 & -5\\ 7 & 16\end{bmatrix}$

$$\begin{bmatrix} 5 & 0 & 2 \\ -2 & 7 & -3 \\ 1 & -3 & 1 \end{bmatrix}\begin{bmatrix} 5 & 1 & 5 \\ -2 & -4 & 3 \\ 3 & 2 & -4 \end{bmatrix} =$$

b) $$\begin{bmatrix} (5)(5)+(0)(-2)+(2)(3) & (5)(1)+(0)(-4)+(2)(4) & (5)(5)+(0)(3)+(2)(-4) \\ (-2)(5)+(7)(-2)+(-3)(3) & (-2)(1)+(7)(-4)+(-3)(4) & (-2)(5)+(7)(3)+(-3)(-4) \\ (1)(5)+(-3)(-2)+(1)(3) & (1)(1)+(-3)(-4)+(1)(4) & (1)(5)+(-3)(3)+(1)(-4) \end{bmatrix} =$$

$$\begin{bmatrix} 25+0+6 & 5+0+8 & 25+0-8 \\ -10-14-9 & -2-28-12 & -10+21+12 \\ 5+6+3 & 1+12+4 & 5-9-4 \end{bmatrix} = \begin{bmatrix} 31 & 13 & 17 \\ -33 & -42 & 23 \\ 14 & 17 & -8 \end{bmatrix}$$

3. Determine the value of the determinant using the indicated method:

a) diagonal method

$$\begin{vmatrix} 4 & 3 & 8 \\ -6 & -5 & 4 \\ 2 & -2 & -1 \end{vmatrix} \begin{matrix} 4 & 3 & 8 & 4 & 3 \\ -6 & -5 & 4 & -6 & -5 \\ 2 & -2 & -1 & 2 & -2 \end{matrix} =$$

$$[(4)(-5)(-1)+(3)(4)(2)+(8)(-6)(-2)]-[(3)(-6)(-1)+(4)(4)(-2)+(8)(-5)(2)]=$$

$$[20+24+96]-[18-32-80]=$$

$$[140]-[-94]=234$$

b) expansion by minors

$$\begin{vmatrix} 7 & -2 & 8 \\ -3 & 1 & 4 \\ 2 & 4 & -5 \end{vmatrix} = (7)\begin{vmatrix} 1 & 4 \\ 4 & -5 \end{vmatrix} - (-2)\begin{vmatrix} -3 & 4 \\ 2 & -5 \end{vmatrix} + (8)\begin{vmatrix} -3 & 1 \\ 2 & 4 \end{vmatrix} =$$

$$(7)\{(1)(-5)-(4)(4)\}+2\{(-3)(-5)-(4)(2)\}+8\{(-3)(4)-(1)(2)\}=$$

$$(7)\{-5-16\}+2\{15-8\}+8\{-12-2\}=$$

$$(7)\{-21\}+2\{7\}+8\{-14\}=-84+14-112=-182$$

c) properties of determinants

$$\begin{vmatrix} 12 & -4 & -7 \\ -8 & 6 & 11 \\ 5 & 13 & -8 \end{vmatrix}$$ multiply column "2" by 3 add to column "1" $\Rightarrow$

$$\begin{vmatrix} 0 & -4 & -7 \\ 10 & 6 & 11 \\ 44 & 13 & -8 \end{vmatrix}$$ multiply column "2" by 2 add to column "3" $\Rightarrow$

$$\begin{vmatrix} 0 & -4 & 1 \\ 10 & 6 & 23 \\ 44 & 13 & 18 \end{vmatrix}$$ multiply column "3" by -4 add to column "2"

$$\begin{vmatrix} 0 & 0 & 1 \\ 10 & -86 & 23 \\ 44 & -59 & 18 \end{vmatrix}$$ Expand by minors on row 1

$$+(1)\begin{vmatrix} 10 & -86 \\ 44 & -59 \end{vmatrix} = 1(10)(-59) - (-86)(44) = -590 + 3784 = 3194$$

d) Solve for "x" using either diagonal method, expansion by minors, zeros

$$\begin{vmatrix} -2 & 4 & -1 \\ 1 & -3 & x \\ 2 & 5 & 1 \end{vmatrix} = 30$$

$$-(1)\begin{vmatrix} 4 & -1 \\ 5 & 1 \end{vmatrix} + (-3)\begin{vmatrix} -2 & -1 \\ 2 & 1 \end{vmatrix} - (x)\begin{vmatrix} -2 & 4 \\ 2 & 5 \end{vmatrix} = 30$$

$$-(1)\left[(4)(1) - (-1)(5)\right] - 3\left[(-2)(1) - (-1)(2)\right] - x[(-2)(5) - (4)(2)] = 30$$

$$-1\left[4 + 5\right] - 3\left[-2 + 2\right] - x\left[-10 - 8\right] = 30$$

$$-9 + 18x = 30$$

$$x = \frac{39}{18}$$

4. Solve:

a) $$3\begin{bmatrix} 4 & 1 \\ -2 & 7 \end{bmatrix} + 5\begin{bmatrix} a & c \\ b & d \end{bmatrix} = -4\begin{bmatrix} -2 & 6 \\ 5 & -4 \end{bmatrix} \Rightarrow \begin{bmatrix} 12 & 3 \\ -6 & 21 \end{bmatrix} + 5\begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} 8 & -24 \\ -20 & 16 \end{bmatrix} \Rightarrow$$

$$5\begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} -4 & -27 \\ -14 & -5 \end{bmatrix} \Rightarrow a = -\frac{4}{5}, b = -\frac{14}{5}, c = -\frac{27}{5}, d = \frac{-5}{5}$$

$5x+2y=9$

$-4x+7y=-4$

b) $\begin{bmatrix}5 & 2\\ -4 & 7\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}9\\ -4\end{bmatrix}\Rightarrow\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{43}\begin{bmatrix}7 & -2\\ 4 & 5\end{bmatrix}\begin{bmatrix}9\\ -4\end{bmatrix}\Rightarrow\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{43}\begin{bmatrix}7(9)+(-2)(-4)\\ 4(9)+(5)(-4)\end{bmatrix}$

$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{43}\begin{bmatrix}71\\ 16\end{bmatrix}\Rightarrow\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}71/43\\ 16/43\end{bmatrix}$

$2\begin{bmatrix}4 & -2\\ 1 & 5\end{bmatrix}-4\begin{bmatrix}3 & 2\\ -x & 1\end{bmatrix}=3\begin{bmatrix}y & -4\\ -5 & 2\end{bmatrix}\Rightarrow\begin{bmatrix}8 & -4\\ 2 & 10\end{bmatrix}+\begin{bmatrix}-12 & -8\\ 4x & -4\end{bmatrix}=\begin{bmatrix}3y & -12\\ -15 & 6\end{bmatrix}\Rightarrow$

c) $8+(-12)=3y\Rightarrow -4=3y\Rightarrow y=-\frac{4}{3}$

$2+4x=-15\Rightarrow 4x=-17\Rightarrow x=-\frac{17}{4}$

d) Use either expansion by minors, diagonal or zeros to determine each of the values.

$4x+2y-3x=-6$

$-2x+y+4z=2$

$x+2y-z=8$

$D=\begin{vmatrix}4 & 2 & -3\\ -2 & 1 & 4\\ 1 & 2 & -1\end{vmatrix}=-17$ $\qquad D_x=\begin{vmatrix}-6 & 2 & -3\\ 2 & 1 & 4\\ 8 & 2 & -1\end{vmatrix}=134$

$D_y=\begin{vmatrix}4 & -6 & -3\\ -2 & 2 & 4\\ 1 & 8 & -1\end{vmatrix}=-94$ $\qquad D_z=\begin{vmatrix}4 & 2 & -6\\ -2 & 1 & 2\\ 1 & 2 & 8\end{vmatrix}=82$

$x=\frac{D_x}{D}=\frac{134}{-17}=-\frac{134}{17}, y=\frac{D_y}{D}=\frac{-94}{-17}=\frac{94}{17}, z=\frac{D_z}{D}=\frac{82}{-17}=-\frac{82}{17}$