Radicals

1. $\sqrt{3a}\cdot\sqrt{5ab}=a\sqrt{3\cdot 5b}$
2. $\dfrac{\sqrt{18x}}{\sqrt{3xy}}=\dfrac{\sqrt{6}}{y}=\dfrac{\sqrt{6y}}{y}$
3. $\sqrt[3]{x^9y^2}=x^3\sqrt[3]{y^2}$
4. $\sqrt{3x^4y^7}=x^2y^3\sqrt{3y}$
5. $\sqrt[3]{2x^2}\cdot\sqrt[3]{4x}=\sqrt[3]{2^3x^3}=2x$
6. $\sqrt{4x^5y^2}\cdot\sqrt{16x^4y^9}=\sqrt{2^6x^9y^{11}}=2^3x^4y^5\sqrt{xy}$
7. $\sqrt[4]{27}\cdot\sqrt{3}=\left(3^3\right)^{1/4}(3)^{1/2}=3^{3/4}3^{1/2}=3^{5/4}=\sqrt[4]{3^5}=3\sqrt[4]{3}$
8. $\dfrac{\sqrt[3]{25}}{\sqrt{5}}=\dfrac{\left(5^2\right)^{1/3}}{5^{1/2}}=\dfrac{5^{2/3}}{5^{1/2}}=5^{1/6}=\sqrt[6]{5}$
9. $\dfrac{\sqrt{6}}{\sqrt[3]{9}}=\dfrac{(2\cdot 3)^{1/2}}{\left(3^2\right)^{1/3}}=\dfrac{2^{1/2}3^{1/2}}{3^{2/3}}=\dfrac{2^{3/6}}{3^{1/6}}=\dfrac{\sqrt[6]{2^3}}{\sqrt[6]{3}}\cdot\dfrac{\sqrt[6]{3^5}}{\sqrt[6]{3^5}}=\dfrac{\sqrt[6]{2^3 3^5}}{3}$
10. $\sqrt[4]{27a^2}\cdot\sqrt[4]{3a^3b}=\left(3^3a^2\right)^{1/4}\left(3a^3b\right)^{1/4}=3^{3/4}a^{2/4}3^{1/4}a^{3/4}b^{1/4}=3^{4/4}a^{5/4}b^{1/4}=3a\sqrt[4]{ab}$
11. $\sqrt[6]{72}\cdot\sqrt[3]{18}=\left(2^3 3^2\right)^{1/6}\left(2\cdot 3^2\right)^{1/3}=2^{3/6}3^{2/6}2^{1/3}3^{2/3}=2^{5/6}3^{6/6}=3\sqrt[6]{2^5}$
12. $\sqrt{2x}\cdot\sqrt[3]{4x^2y}\cdot\sqrt[6]{16x^3y}=(2x)^{1/2}\left(2^2x^2y\right)^{1/3}\left(2^4x^3y\right)^{1/6}=2^{1/2}x^{1/2}2^{2/3}x^{2/3}y^{1/3}2^{4/6}x^{3/6}y^{1/6}=$
$2^{3/6}2^{4/6}2^{4/6}x^{3/6}x^{4/6}x^{3/6}y^{2/6}y^{1/6}=2^{11/6}x^{10/6}y^{3/6}=2x\sqrt[6]{2^5x^4y^3}$
13. $\sqrt{10x}\cdot\sqrt[4]{40x^3}=(2\cdot 5x)^{1/2}\left(2^3 5x^3\right)^{1/4}=2^{1/2}5^{1/2}x^{1/2}2^{3/4}5^{1/4}x^{3/4}=2^{2/4}2^{3/4}5^{2/4}5^{1/4}x^{2/4}x^{3/4}=$
$2^{5/4}5^{3/4}x^{5/4}=2x\sqrt[4]{2\cdot 5^3x}$
14. $\left(3+2\sqrt{3}\right)\left(3-2\sqrt{3}\right)=9-4\sqrt{3^2}=9-12=-3$
15. $\left(\sqrt[3]{4}-\sqrt[3]{9}\right)^2=\left(\sqrt[3]{2^2}-\sqrt[3]{3^2}\right)^2=\left(\sqrt[3]{2^2}-\sqrt[3]{3^2}\right)\left(\sqrt[3]{2^2}-\sqrt[3]{3^2}\right)=$
$\sqrt[3]{2^4}-\sqrt[3]{2^2 3^2}-\sqrt[3]{2^2 3^2}+\sqrt[3]{3^4}=2\sqrt[3]{2}-2\sqrt[3]{2^2 3^2}+3\sqrt[3]{3}$
16. $\left(3x^2y^4\right)^{\frac{1}{2}}=\sqrt{3x^2y^4}=xy^2\sqrt{3}$
17. $\left(8x^4y^9\right)^{\frac{3}{2}}(4xy)^{\frac{1}{2}}=\left(2^3x^4y^9\right)^{\frac{3}{2}}\left(2^2xy\right)^{\frac{1}{2}}=2^{9/2}x^{12/2}y^{27/2}2^{2/2}x^{1/2}y^{1/2}=2^{11/2}x^{13/2}y^{28/2}=2^5x^6y^{14}\sqrt{2x}$
18. $\left(x^{\frac{1}{2}}+2y^{-\frac{1}{2}}\right)\left(x^{\frac{1}{2}}-2y^{-\frac{1}{2}}\right)=x-2x^{1/2}y^{-1/2}+2x^{1/2}y^{-1/2}-4y^{-1}=x-\dfrac{4}{y}$
19. $\sqrt[3]{x^9y^2}=x^3\sqrt[3]{y^2}$
20. $\sqrt{x^4y^5}\cdot\sqrt[3]{x^7y^9}=\left(x^4y^5\right)^{1/2}\left(x^7y^9\right)^{1/3}=x^{4/2}y^{5/2}x^{7/3}y^{9/3}=x^{12/6}x^{14/6}y^{15/6}y^{18/6}=x^{26/6}y^{33/6}=x^4y^5\sqrt[6]{x^2y^3}$

21. $\left(8x^4y^{-3}\right)^{-\frac{1}{2}} = \left(2^3x^4y^{-3}\right)^{-1/2} = 3^{-3/2}x^{-4/2}y^{3/2} = \dfrac{y^{3/2}}{3^{3/2}x^{4/2}} = \dfrac{\sqrt{y^3}}{\sqrt{3^3x^4}} = \dfrac{y\sqrt{y}3}{3^2x^2}$

22. $\sqrt{30x}\cdot\sqrt[4]{24x^2}\cdot\sqrt[3]{75x} = \left(2\cdot3\cdot5x\right)^{1/2}\left(2^3\cdot3x^2\right)^{1/4}\left(3\cdot5^2x\right)^{1/3} = 2^{1/2}3^{1/2}5^{1/2}x^{1/2}2^{3/4}3^{1/4}x^{2/4}3^{1/3}5^{2/3}x^{1/3} =$
$2^{6/12}2^{9/12}3^{6/12}3^{2/12}3^{4/12}5^{6/12}5^{8/12}x^{6/12}x^{6/12} = 2^{15/12}3^{13/12}5^{14/12}x^{16/12} = 2\cdot3\cdot5x\sqrt[12]{2^3\cdot3\cdot5^2x^4}$

23. $\sqrt{5xy}\cdot\sqrt[3]{100x^2} = \left(5xy\right)^{1/2}\left(2^25^2x^2\right)^{1/3} = 5^{1/2}x^{1/2}y^{1/2}2^{2/3}5^{2/3}x^{2/3} = 5^{3/6}x^{3/6}y^{3/6}2^{4/6}5^{4/6}x^{4/6} =$
$2^{4/6}5^{7/6}x^{7/6}y^{3/6} = 5x\sqrt[6]{2^4\cdot5xy^3}$

24. $\left(\sqrt[5]{\dfrac{16x^3}{24y^4}}\right)^2 = \left(\left(\dfrac{2^4x^3}{2^3\cdot3y^4}\right)^{\frac{1}{5}}\right)^2 = \left(\dfrac{2x^3}{3y^4}\right)^{2/5} = \dfrac{2^{2/5}x^{6/5}}{3^{2/5}y^{8/5}} = \dfrac{\sqrt[5]{2^2x^6}}{\sqrt[5]{3^2y^8}}\cdot\dfrac{\sqrt[5]{3^3y^2}}{\sqrt[5]{3^3y^2}} = \dfrac{x\sqrt[5]{2^2\cdot3^3xy^2}}{3y^2}$

25. $\sqrt[4]{2\sqrt[3]{16x^3y}} = \left(2\left(2^4x^3y\right)^{1/3}\right)^{1/4} = \left(2\cdot2^{4/3}x^{3/3}y^{1/3}\right)^{1/4} = 2^{1/4}2^{4/12}x^{3/12}y^{1/12} = 2^{7/12}x^{3/12}y^{1/12} = \sqrt[12]{2^7x^3y^1}$

26. $\sqrt[3]{2x^3y\sqrt{8x^3y^5}} = \left(2x^3y\left(2^3x^3y^5\right)^{1/2}\right)^{1/3} = \left(2x^3y\cdot2^{3/2}x^{3/2}y^{5/2}\right)^{1/3} = 2^{1/3}x^{3/3}y^{1/3}2^{3/6}x^{3/6}y^{5/6} =$
$2^{5/6}x^{9/6}y^{7/6} = \sqrt[6]{2^5x^9y^7} = xy\sqrt[6]{2^5x^3y}$

27. $\dfrac{\sqrt{2x}\cdot\sqrt[3]{4x^2y^2}}{\sqrt[6]{16x^3y^4}} = \dfrac{\left(2x\right)^{1/2}\left(2^2x^2y^2\right)^{1/3}}{\left(2^4x^3y^4\right)^{1/6}} = \dfrac{2^{1/2}x^{1/2}2^{2/3}x^{2/3}y^{2/3}}{2^{4/6}x^{3/6}y^{4/6}} = \dfrac{2^{3/6}2^{4/6}x^{3/6}x^{4/6}y^{4/6}}{2^{4/6}x^{3/6}y^{4/6}} = 2^{3/6}x^{4/6} = \sqrt[6]{2^3x^4}$

28. $\dfrac{5x\sqrt{x}}{\sqrt[4]{50x^3y^2}\cdot\sqrt[3]{20x^2y}} = \dfrac{5x\cdot x^{1/2}}{\left(2\cdot5^2x^3y^2\right)^{1/4}\left(2^2\cdot5x^2y\right)^{1/3}} = \dfrac{5x\cdot x^{1/2}}{2^{1/4}5^{2/4}x^{3/4}y^{2/4}2^{2/3}5^{1/3}x^{2/3}y^{1/3}} =$
$\dfrac{5^{12/12}x^{12/12}x^{6/12}}{2^{3/12}2^{8/12}5^{6/12}5^{4/12}x^{9/12}x^{8/12}y^{6/12}y^{4/12}} = \dfrac{5^{2/12}x^{1/12}}{2^{11/12}y^{10/12}} = \dfrac{\sqrt[12]{5^2x}}{\sqrt[12]{2^{11}y^{10}}}\cdot\dfrac{\sqrt[12]{2y^2}}{\sqrt[12]{2y^2}} = \dfrac{\sqrt[12]{2\cdot5^2xy^2}}{2y}$

29. $\dfrac{12x\sqrt[3]{4x^2}}{\sqrt{2x}\cdot\sqrt[6]{8x^4}} = \dfrac{2^2\cdot3x\left(2^2x^2\right)^{1/3}}{\left(2x\right)^{1/2}\left(2^3x^4\right)^{1/6}} = \dfrac{2^2\cdot3x\cdot2^{2/3}x^{2/3}}{2^{1/2}x^{1/2}2^{3/6}x^{4/6}} = \dfrac{2^{12/6}2^{4/6}3^{6/6}x^{6/6}x^{4/6}}{2^{3/6}2^{3/6}x^{3/6}x^{4/6}} = 2^{10/6}3^{6/6}x^{3/6} =$
$\sqrt[6]{2^{10}3^6x^3} = 2\cdot3\sqrt[6]{2^4x^3}$

30. $\sqrt[3]{-5xy^2}\cdot\sqrt[3]{-75x^5y} = \left(-5xy^2\cdot-3\cdot5^2x^5y\right)^{1/3} = \left(3\cdot5^3x^6y^3\right)^{1/3} = \sqrt[3]{3\cdot5^3x^6y^3} = 5x^2y\sqrt[3]{3}$

31. $\sqrt[3]{\dfrac{4x\sqrt{y^3}}{6x^2\sqrt{y}}}=\left(\dfrac{2^2xy^{1/3}}{2\cdot 3x^2y^{1/2}}\right)^{1/3}=\dfrac{2^{2/3}x^{1/3}y^{1/9}}{2^{1/3}3^{1/3}x^{2/3}y^{1/6}}=\dfrac{2^{12/18}x^{6/18}y^{2/18}}{2^{6/18}3^{6/18}x^{12/18}y^{3/18}}=\dfrac{2^{6/18}}{3^{6/18}x^{6/18}y^{1/18}}=$
$\dfrac{\sqrt[18]{2^6}}{\sqrt[18]{3^6x^6y}}\cdot\dfrac{\sqrt[18]{3^{12}x^{12}y^{17}}}{\sqrt[18]{3^{12}x^{12}y^{17}}}=\dfrac{\sqrt[18]{2^63^{12}x^{12}y^{17}}}{3xy}$

32. $\sqrt{\dfrac{\sqrt[3]{25x^2y^3}}{\sqrt{5xy}}}=\left(\dfrac{\left(5^2x^2y^3\right)^{1/3}}{(5xy)^{1/2}}\right)^{1/2}=\dfrac{5^{2/6}x^{2/6}y^{3/6}}{5^{1/2}x^{1/2}y^{1/2}}=\dfrac{5^{2/6}x^{2/6}y^{3/6}}{5^{3/6}x^{3/6}y^{3/6}}=\dfrac{1}{5^{1/6}x^{1/6}}=\dfrac{1}{\sqrt[6]{5x}}\cdot\dfrac{\sqrt[6]{5^5x^5}}{\sqrt[6]{5^5x^5}}=\dfrac{\sqrt[6]{5^5x^5}}{5x}$

33. $\sqrt[5]{\dfrac{25x^4y^2}{\sqrt{625x^3y^4}}}=\left(\dfrac{5^2x^4y^2}{\left(5^4x^3y^4\right)^{1/2}}\right)^{1/5}=\dfrac{5^{2/5}x^{4/5}y^{2/5}}{5^{4/10}x^{3/10}y^{4/10}}=\dfrac{5^{4/10}x^{8/10}y^{4/10}}{5^{4/10}x^{3/10}y^{4/10}}=x^{5/10}=\sqrt[10]{x^5}$

34. $\dfrac{\sqrt{5x^3y^4}\cdot\sqrt[3]{25xy^{-3}}}{\sqrt[5]{125x^{-4}y^2}}=\dfrac{\left(5x^3y^4\right)^{1/2}\left(5^2xy^{-3}\right)^{1/3}}{\left(5^3x^{-4}y^2\right)^{1/5}}=\dfrac{5^{1/2}x^{3/2}y^{4/2}5^{2/3}x^{1/3}y^{-3/3}}{5^{3/5}x^{-4/5}y^{2/5}}=\dfrac{5^{15/30}5^{20/30}x^{45/30}x^{10/30}x^{24/30}y^{60/30}}{5^{18/30}y^{12/30}y^{30/30}}=$
$5^{17/30}x^{79/30}y^{18/30}=\sqrt[30]{5^{17}x^{79}y^{18}}=x^2\sqrt[30]{5^{17}x^{19}y^{18}}$

35. $\dfrac{\sqrt[4]{x^5y^7z^{-3}}\cdot\left(\sqrt[3]{x^{-2}y^4}\right)^{-3}\cdot\sqrt[5]{y^{-5}z^{-10}}}{\sqrt[20]{x^{19}y^{-23}}\cdot\left(\sqrt[6]{x^{-5}y^2z^{42}}\right)^2}=\dfrac{\left(x^5y^7z^{-3}\right)^{1/4}\left(\left(x^{-2}y^4\right)^{1/3}\right)^{-3}\left(y^{-5}z^{-10}\right)^{1/5}}{\left(x^{19}y^{-23}\right)^{1/20}\left(\left(x^{-5}y^2z^{42}\right)^{1/6}\right)^2}=\dfrac{x^{5/4}y^{7/4}z^{-3/4}x^{6/3}y^{-12/3}y^{-5/5}z^{-10/5}}{x^{19/20}y^{-23/20}x^{-10/6}y^{4/6}z^{84/6}}=$
$\dfrac{x^{75/60}x^{120/60}x^{100/60}y^{105/60}y^{69/60}}{x^{57/60}y^{240/60}y^{60/60}y^{40/60}z^{840/60}z^{45/60}z^{120/60}}=\dfrac{x^{238/60}}{y^{166/60}z^{1005/60}}=\dfrac{\sqrt[60]{x^{238}}}{\sqrt[60]{y^{166}z^{1005}}}\cdot\dfrac{\sqrt[60]{y^{14}z^{15}}}{\sqrt[60]{y^{14}z^{15}}}=\dfrac{x^3\sqrt[60]{x^{58}y^{14}z^{15}}}{y^3z^{17}}$