Laws of Exponents

Given: $\sqrt[n]{x} = x^{\frac{1}{n}}, \quad (\sqrt[n]{x})^m = x^{\frac{m}{n}}, \quad \sqrt[n]{x^m} = x^{\frac{m}{n}}, \quad \sqrt[n]{\frac{x}{y}} = \frac{x^{\frac{1}{n}}}{y^{\frac{1}{n}}}, \quad \left(\sqrt[n]{\frac{x^m}{y^p}}\right) = \frac{x^{\frac{m}{n}}}{y^{\frac{p}{n}}}$

Write each of the following radicals in simplified exponential form and simplified radical form:

Simplified exponential first and simplified radical after the arrow.

1. $\sqrt{x} = x^{\frac{1}{2}} \Rightarrow \sqrt{x}$

2. $\sqrt{x^3} = x^{\frac{3}{2}} \Rightarrow \sqrt{x^3} = x\sqrt{x}$

3. $\sqrt{xy} = (xy)^{\frac{1}{2}} = x^{\frac{1}{2}}y^{\frac{1}{2}} \Rightarrow \sqrt{xy}$

4. $\sqrt{x^3y^5} = \left(x^3y^5\right)^{\frac{1}{2}} = x^{\frac{3}{2}}y^{\frac{5}{2}} \Rightarrow \sqrt{x^3y^5} = xy^2\sqrt{xy}$

5. $\sqrt{x^{11}y^2} = \left(x^{11}y^2\right)^{\frac{1}{2}} = x^{\frac{11}{2}}y^{\frac{2}{2}} \Rightarrow \sqrt{x^{11}y^2} = x^5y\sqrt{x}$

6. $\sqrt[3]{x} = x^{\frac{1}{3}} \Rightarrow \sqrt[3]{x}$

7. $\sqrt[4]{x^2y^5} = \left(x^2y^5\right)^{\frac{1}{4}} = x^{\frac{2}{4}}y^{\frac{5}{4}} \Rightarrow \sqrt[4]{x^2y^5} = y\sqrt[4]{x^2y}$

8. $\sqrt[5]{x^5y^3z^{12}} = \left(x^5y^3z^{12}\right)^{\frac{1}{5}} = x^{\frac{5}{5}}y^{\frac{3}{5}}z^{\frac{12}{5}} \Rightarrow \sqrt[5]{x^5y^3z^{12}} = xz^2\sqrt[5]{y^3z^2}$

9. $\sqrt{\frac{x}{y}} = \frac{x^{\frac{1}{2}}}{y^{\frac{1}{2}}} \Rightarrow \sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}} = \frac{\sqrt{x}}{\sqrt{y}} \cdot \frac{\sqrt{y}}{\sqrt{y}} = \frac{\sqrt{xy}}{y}$

10. $\sqrt{\frac{x^3}{y^7}} = \frac{x^{\frac{3}{2}}}{y^{\frac{7}{2}}} \Rightarrow \sqrt{\frac{x^3}{y^7}} = \frac{\sqrt{x^3}}{\sqrt{y^7}} = \frac{\sqrt{x^3}}{\sqrt{y^7}} \cdot \frac{\sqrt{y}}{\sqrt{y}} = \frac{x\sqrt{xy}}{y^4}$

11. $\sqrt[5]{\frac{x^5y^9}{z^7}} = \frac{x^{\frac{5}{5}}y^{\frac{9}{5}}}{z^{\frac{7}{5}}} \Rightarrow \sqrt[5]{\frac{x^5y^9}{z^7}} = \frac{\sqrt[5]{x^5y^9}}{\sqrt[5]{z^7}} = \frac{\sqrt[5]{x^5y^9}}{\sqrt[5]{z^7}} \cdot \frac{\sqrt[5]{z^3}}{\sqrt[5]{z^3}} = \frac{xy\sqrt[5]{y^4z^3}}{z^2}$

12. $\sqrt{x}\sqrt[3]{x} = x^{\frac{1}{2}} \cdot x^{\frac{1}{3}} = x^{\frac{3}{6}} \cdot x^{\frac{2}{6}} = x^{\frac{5}{6}} \Rightarrow \sqrt[6]{x^5}$

13. $\sqrt{xy^2}\sqrt[4]{x^3y^5} = \left(xy^2\right)^{\frac{1}{2}} \cdot \left(x^3y^5\right)^{\frac{1}{4}} = x^{\frac{1}{2}}y^{\frac{2}{2}} \cdot x^{\frac{3}{4}}y^{\frac{5}{4}} = x^{\frac{2}{4}}y^{\frac{4}{4}} \cdot x^{\frac{3}{4}}y^{\frac{5}{4}} = x^{\frac{5}{4}}y^{\frac{9}{4}} \Rightarrow \sqrt[4]{x^5y^9} = xy^2\sqrt[4]{xy}$

14. $\dfrac{\sqrt{x}}{\sqrt[3]{x}} = \dfrac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}} = \dfrac{x^{\frac{3}{6}}}{x^{\frac{2}{6}}} = x^{\frac{1}{6}} \Rightarrow \sqrt[6]{x}$

15. $\dfrac{\sqrt{x^5}}{\sqrt[5]{x^2}} = \dfrac{x^{\frac{5}{2}}}{x^{\frac{2}{5}}} = \dfrac{x^{\frac{25}{10}}}{x^{\frac{4}{10}}} = x^{\frac{21}{10}} \Rightarrow \sqrt[10]{x^{21}} = x^2\sqrt[10]{x}$

16. $\dfrac{\sqrt[3]{x^2y}}{\sqrt{x^3y^7}} = \dfrac{\left(x^2y\right)^{\frac{1}{3}}}{\left(x^3y^7\right)^{\frac{1}{2}}} = \dfrac{x^{\frac{2}{3}}y^{\frac{1}{3}}}{x^{\frac{3}{2}}y^{\frac{7}{2}}} = \dfrac{x^{\frac{4}{6}}y^{\frac{2}{6}}}{x^{\frac{9}{6}}y^{\frac{21}{6}}} = \dfrac{1}{x^{\frac{5}{6}}y^{\frac{19}{6}}} \Rightarrow \dfrac{1}{\sqrt[6]{x^5y^{19}}} = \dfrac{1}{\sqrt[6]{x^5y^{19}}} \cdot \dfrac{\sqrt[6]{xy^5}}{\sqrt[6]{xy^5}} = \dfrac{\sqrt[6]{xy^5}}{xy^4}$

17. $\left(\sqrt{x^3}\right)^5 = \left(x^{\frac{3}{2}}\right)^{\frac{1}{5}} = x^{\frac{3}{10}} \Rightarrow \sqrt[10]{x^3}$

18. $\left(\sqrt{x^3y^5}\right)^4 = \left(\left(x^3y^5\right)^{\frac{1}{2}}\right)^4 = \left(x^{\frac{3}{2}}y^{\frac{5}{2}}\right)^4 = x^{\frac{12}{2}}y^{\frac{20}{2}} \Rightarrow \sqrt{x^{12}y^{20}} = x^6y^{10}$

19. $\dfrac{\left(\sqrt{xy^3}\right)^5}{\left(\sqrt[3]{x^2y}\right)^2} = \dfrac{\left(\left(xy^3\right)^{\frac{1}{2}}\right)^5}{\left(\left(x^2y\right)^{\frac{1}{3}}\right)^2} = \dfrac{\left(x^{\frac{1}{2}}y^{\frac{3}{2}}\right)^5}{\left(x^{\frac{2}{3}}y^{\frac{1}{3}}\right)^2} = \dfrac{x^{\frac{5}{2}}y^{\frac{15}{2}}}{x^{\frac{4}{3}}y^{\frac{2}{3}}} = \dfrac{x^{\frac{15}{6}}y^{\frac{45}{6}}}{x^{\frac{8}{6}}y^{\frac{4}{6}}} = x^{\frac{7}{6}}y^{\frac{41}{6}} \Rightarrow \sqrt[6]{x^7y^{41}} = xy^6\sqrt[6]{xy^5}$

20. $\left(\sqrt[3]{x^2y^3}\right)^5\left(\sqrt[4]{xy^5}\right)^2 = \left(\left(x^2y^3\right)^{\frac{1}{3}}\right)^5 \cdot \left(\left(xy^5\right)^{\frac{1}{4}}\right)^2 = \left(x^{\frac{2}{3}}y^{\frac{3}{3}}\right)^5 \cdot \left(x^{\frac{1}{4}}y^{\frac{5}{4}}\right)^2 = x^{\frac{10}{3}}y^{\frac{15}{3}} \cdot x^{\frac{2}{4}}y^{\frac{10}{4}} =$
$x^{\frac{40}{12}}y^{\frac{60}{12}} \cdot x^{\frac{6}{12}}y^{\frac{30}{12}} = x^{\frac{46}{12}}y^{\frac{90}{12}} \Rightarrow \sqrt[12]{x^{46}y^{90}} = x^3y^7\sqrt[12]{x^{10}y^6}$

21. $\dfrac{\left(\sqrt{xy}\right)^3\left(\sqrt[3]{x^3y^2}\right)^2}{\sqrt[4]{x^4y^3}} = \dfrac{\left(\left(xy\right)^{\frac{1}{2}}\right)^3\left(\left(x^3y^2\right)^{\frac{1}{3}}\right)^2}{\left(x^4y^3\right)^{\frac{1}{4}}} = \dfrac{\left(x^{\frac{1}{2}}y^{\frac{1}{2}}\right)^3\left(x^{\frac{3}{3}}y^{\frac{2}{3}}\right)^2}{\left(x^4y^3\right)^{\frac{1}{4}}} = \dfrac{x^{\frac{3}{2}}y^{\frac{3}{2}} \cdot x^{\frac{6}{3}}y^{\frac{4}{3}}}{x^{\frac{4}{4}}y^{\frac{3}{4}}} = \dfrac{x^{\frac{18}{12}}y^{\frac{18}{12}} \cdot x^{\frac{24}{12}}y^{\frac{16}{12}}}{x^{\frac{12}{12}}y^{\frac{9}{12}}} =$
$\dfrac{x^{\frac{42}{12}}y^{\frac{34}{12}}}{x^{\frac{12}{12}}y^{\frac{9}{12}}} = x^{\frac{30}{12}}y^{\frac{25}{12}} \Rightarrow \sqrt[12]{x^{30}y^{25}} = x^2y^2\sqrt[12]{x^6y}$

22. $\sqrt[4]{\sqrt[3]{\sqrt{x^5y^7}}} = \left(\left(\left(x^5y^7\right)^{\frac{1}{2}}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}} = x^{\frac{5}{24}}y^{\frac{7}{24}} \Rightarrow \sqrt[24]{x^5y^7}$

23. $\sqrt[3]{\sqrt[4]{\sqrt{256x^{34}y^{29}}}} = \left(\left(\left(2^8x^{34}y^{29}\right)^{\frac{1}{2}}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}} = 2^{\frac{8}{24}}x^{\frac{34}{24}}y^{\frac{29}{24}} \Rightarrow \sqrt[24]{2^8x^{34}y^{29}} = xy\sqrt[24]{2^8x^{10}y^5}$

$$\frac{\sqrt{12x^3y^5}\sqrt[4]{24x^{-5}y^3}}{\sqrt[3]{36x^4y^{-4}}} = \frac{\left(2^2\cdot 3x^3y^5\right)^{\frac{1}{2}}\left(2^3\cdot 3x^{-5}y^3\right)^{\frac{1}{4}}}{\left(2^2\cdot 3^2x^4y^{-4}\right)^{\frac{1}{3}}} = \frac{2^{\frac{2}{2}}\cdot 3^{\frac{1}{2}}x^{\frac{3}{2}}y^{\frac{5}{2}}\cdot 2^{\frac{3}{4}}\cdot 3^{\frac{1}{4}}x^{\frac{-5}{4}}y^{\frac{3}{4}}}{2^{\frac{2}{3}}\cdot 3^{\frac{2}{3}}x^{\frac{4}{3}}y^{\frac{-4}{3}}} =$$

24. $$\frac{2^{\frac{12}{12}}\cdot 3^{\frac{6}{12}}x^{\frac{18}{12}}y^{\frac{30}{12}}\cdot 2^{\frac{9}{12}}\cdot 3^{\frac{3}{12}}y^{\frac{9}{12}}y^{\frac{16}{12}}}{2^{\frac{8}{12}}\cdot 3^{\frac{8}{12}}x^{\frac{15}{12}}x^{\frac{16}{12}}} = \frac{2^{\frac{21}{12}}3^{\frac{9}{12}}x^{\frac{18}{12}}y^{\frac{55}{12}}}{2^{\frac{8}{12}}\cdot 3^{\frac{8}{12}}x^{\frac{31}{12}}} = \frac{2^{\frac{13}{12}}3^{\frac{1}{12}}y^{\frac{55}{12}}}{x^{\frac{13}{12}}} \Rightarrow$$

$$\frac{\sqrt[12]{2^{13}3y^{55}}}{\sqrt[12]{x^{13}}} = \frac{\sqrt[12]{2^{13}3y^{55}}}{\sqrt[12]{x^{13}}}\cdot\frac{\sqrt[12]{x^{11}}}{\sqrt[12]{x^{11}}} = \frac{2y^4\sqrt[12]{2\cdot 3\cdot x^{11}y^7}}{x^2}$$