Arithmetic and Geometric Sequences and Series

1. Determine the indicated term if:
   a) a = -3, n = 7 and d = -2
   b) a = 5, r = -2 and n = 5
2. Find the missing term:
   a) "n" if a = 5, d = 3 and l = 56
   b) "n" if a = 18, r = -3 and $t_n = -4374$
   c) "a" if d = 6, n = 11 and l = -55
   d) "a" if r = -3, n = 7 and l = 2916
   e) "d" if a = 7, n = 13 and l = 43
   f) "r" if a = 5, n = 9 and l = 327680
3. Find the sum of the following:
   a) 4 + 12 + 20 + … to 12 terms
   b) 3 - 15 + 75 - … to 8 terms
   c) for the arithmetic series a = 5, n = 20 and l = 119
   d) a = -2, r = 6 and l = -93312
4. Find the missing terms:
   a) a = 5, r = -2, $S_n = -25$
   b) n = 12, l = 1, $S_n = -24$
5. a) if the 3rd term in an A.P. is 19 and the 8th term is 54, find "a" and the sum of the eight terms.
   b) if the second term is a G.P. is -10 and the 6th term is -6250, find the value of "a" and the sum of the first eight terms.
6. a) Write in summation notation:
   1) 3 + 7 + 11 + 15 + 19 + 23 + 27 + 31 + 35
   2) -5 + 15 - 45 + 135 - … to 15 terms
   b) Expand and find the sum:
   1. $\sum_{n=1}^{9} 3+(n-1)(-6)$
   2. $\sum_{n=4}^{7} 4(-2)^{n-1}$
7. Re-write as a fraction:
   1. $0.\overline{415}$
   2. $3.2\overline{12}$
8. a) Find the sum of the following series:
   12 + 6 + 3 + … to infinity
   b) If the sum of an infinite geometric series is 124 and the value of the first term is 16, what is the common ratio?
9. Problems:
   a) If you had invested $1000 on January 2nd, 2000 and your investment brought a return of 7% per year compounded annually, determine the amount of money would would have in your account on January 2nd, 2008.
   b) If your starting salary was $30,000 and you received yearly increases of $2000;
      1. How much money would you be earning in your 12th year of employment?

2. How many years would it take so that the total amount earned would exceed one million dollars?

c) If a ball is dropped from a height of 150 meters and it rebounds 3/5 the distance it has traveled, determine:
a) the distance traveled in completing the 7th down and up path?
b) distance traveled in completing a total of 7 down and up paths?
c) the distance the ball traveled in coming to rest?

d) The population of a town is decreasing at a rate of 8% per year. What will the population of the town be in 10 years if the present day population is 5000.

10. a) Insert a geometric mean between 7 and 24
b) Insert an arithmetic mean between -9 and 36
c) Insert four geometric means between 6 and 30
d) Insert 5 arithmetic means between -3 and 32

Matrices:

Given: $A=\begin{bmatrix}2 & -3\\ 5 & 4\end{bmatrix}, B=\begin{bmatrix}-6 & 5\\ -1 & 2\end{bmatrix}, C=\begin{bmatrix}-3 & 4\\ -2 & 1\end{bmatrix}$

Calculate: 1. A + B
2. 3B - 2C
3. 5(C - A)
4. A * B
5. B(A + C)
6. A^T
7. B^{-1}
8. Additive inverse of A
9. Multiplicative inverse of C
10. detB

B. Multiply

1. $\begin{bmatrix}2 & -1 & 0\\ -2 & 3 & 1\\ 1 & 2 & 3\end{bmatrix}\bullet\begin{bmatrix}0 & 3 & -1\\ 2 & 1 & 1\\ -1 & 4 & -1\end{bmatrix}$

2. $\begin{bmatrix}4 & 1 & -2\\ 3 & -1 & 4\end{bmatrix}\bullet\begin{bmatrix}1 & 3\\ -1 & -2\\ 2 & 4\end{bmatrix}$

C. Solve the following

1. $2\begin{bmatrix}3 & -x\\ -1 & 5\end{bmatrix}-4\begin{bmatrix}5 & -1\\ y & 3\end{bmatrix}=\begin{bmatrix}4 & -2\\ 7 & -3\end{bmatrix}$

2. $\begin{bmatrix}-2 & 1\\ 3 & -4\end{bmatrix}\bullet A=\begin{bmatrix}5 & -2\\ -1 & 4\end{bmatrix}$

3. 3x - 4y = -2
-x + 5y = -3

Logs and Exponents

Simplify

1. $x^2y^4 \cdot x^5y^2$

2. $(5x^2y^{-3})^4$

3. $\dfrac{25x^4y^2 \cdot 15x^{-5}y^3}{(5x^{-2}y^4)^2}$

4. $\sqrt{3xy} \cdot \sqrt[3]{3^2x^5y^{11}}$

5. $\dfrac{\sqrt[3]{16x^7y^5}}{\sqrt[4]{32x^2y^4}}$

6. $5^{3x=1} \cdot 5^{7x-2}$

7. $(5^{\frac{1}{2}} + 3^{\frac{2}{3}})(5^{\frac{1}{2}} - 3^{\frac{2}{3}})$

8. $(5^{2i} - 2)^2$

9. $125^{\frac{2}{3}}$

Solve the following exponential equations:

1. $3^{2x-1} = 3^{5x+2}$

2. $25^{x-1} = 125^{2x+3}$

3. $5^x = 17$

4. $21 \cdot 5^{3x} = 57$

5. $4^{x^2} = 16^{5x-1}$

6. $\left(\dfrac{1}{3}\right)^{2x} = 9^{x+5}$

Write as log equations and do not solve:

1. $35 \cdot 7^x = \sqrt{17}$

2. $3^x \cdot \sqrt[7]{5} = x^5$

3. $3^{x-1} = 5^{2x-3}$

Solve the following log equations:

1. $\log_3 x = 7$

2. $\log_x 3 = 7$

3. $\log_3 7 = x$

4. $\log_5(x-1) + \log_5(x-3) = \log_5 8$

5. $5\log x - 3\log x = \log 16$

6. $\log_3 x + 2\log_3 x = 5$

7. $\log_x(x^3 - 1) - \log_2(x^2 + x + 1) = 4$

8. $\log_3 5 = \log_2 x$

Problems

1. Determine the number of years it would take for a savings account of $2000 to triple if the money is invested at 8.5% compounded continuously.
2. Determine the half life of a substance is after 400 years 3 kg remain from a mass of 50 kg.
3. Determine how many years are required for a town's population to decrease from 5000 individuals to 1000 if the rate of decrease is 8%.

Quadratic Equations:

1. Given: a) $x^2 - 7x + 1 = 0$
   b) $-5x^2 + 3x + 11 = 0$

   Determine : a) sum of the roots
   b) product of the roots
   c) value of the discriminant
   d) nature of the roots
   e) the roots of the equation

2. Solve the following quadratic equations:

   a) $x + 3\sqrt{x} - 10 = 0$ b) $x^4 + 3x^2 - 6 = 0$

4. Determine the quadratic equations given the following roots:

   a) $\{-3,5\}$ b) $\{3 \pm 5i\}$

5. Graph the following on a number line using a sign chart:

   a) $(x-1)(x+3)(x-4) \geq 0$ b) $3x^2 - x < 4$

Complex Numbers

A. Simplify

1. i^{107} 2. $i^{10} \cdot i^{23} \cdot i^{-15}$ 3. $\frac{1}{i^7} \cdot \frac{1}{i^{20}} \cdot \frac{1}{i^3}$

4. $(5^{2i+3})^{i-2}$ 5. $(3i-7)(2i+5)$ 6. $4i - 7 + 2i + 5 - 6i + 11$

7. $5i^3(2i^7 - 5)$ 8. $\frac{2}{i-5}$ 9. $\frac{3i-2}{2i+5}$

B. Solve

1. $3x + 2i - 5 = 7x + 3 - 7i$ 2. $4xi - 2 = 7xi - 9$

3. $2x + 5 - 7i = 5xi + 3 + 2i$ 4. $2x - 4yi + 2 = 5y + 3xi - 3i$

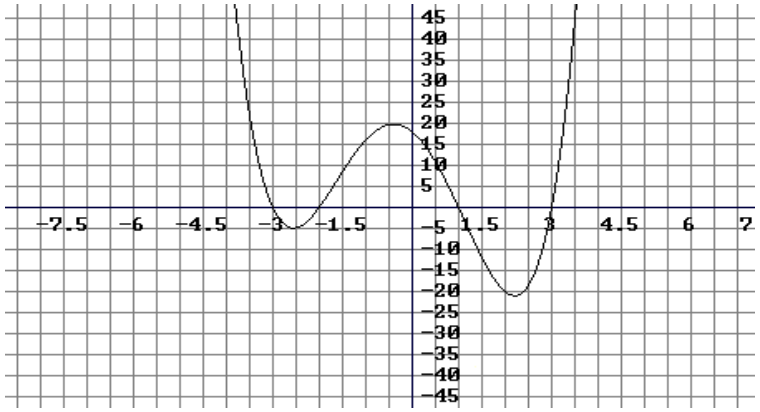
Polynomials

Given: 1. $f(x) = x^4 - 3x^3 - 3x^2 + 11x - 6$
2. $f(x) = x^5 - 4x^4 + x^3 + 10x^2 - 4x - 8$

Determine:
1. value of leading coefficient
2. degree of the function
3. y-intercept
4. x-intercepts

5. multiplicity of zeros
6. where the graph starts and finishes
7. number of hills and the number of values
8. the number of times the graph changes direction
9. the number of positive, negative and imaginary roots
10. presence of holes

2) Given:



Determine:
1. y-intercept
2. x-intercepts (critical zeros)
3. multiplicity of factors
4. where the graph starts and where it finishes
5. factors containing x-intercepts
6. equation of the function
7. possible degree of the function
8. number of hills and number of valleys
9. number of times the graph changes direction
10. number of positive, negative and imaginary roots

Given: $f(x) = \dfrac{2x-7}{x+5}$

Determine:
1. x-intercept
2. y-intercept
3.vertical asymptotes (holes)
4. sketch the graph

Inverse and reciprocals:

1. Write the inverse and reciprocal for: a) $3y = 2x - 1$; b) $y = \dfrac{x-5}{x+3}$

2. Sketch the inverse and reciprocal for: