Rational expression and Equations Answer Key

A. Reduce the following expressions:

1. $\frac{3(x+4y)}{2(x+4y)} = \frac{3}{2}$

2. $\frac{16a-22}{2} = \frac{2(8a-11)}{2} = (8a-11)$

3. $\frac{z^2-9}{z^2+5z+6} = \frac{(z-3)(z+3)}{(z+3)(z+2)} = \frac{(z-3)}{(z+2)}$

4. $\frac{3x^2+4x+1}{2x^2+5x+3} = \frac{(3x+1)(x+1)}{(2x+3)(x+1)} = \frac{(3x+1)}{(2x+3)}$

5. $\frac{x^2+2xy+y^2}{(x+y)^2} = \frac{(x+y)(x+y)}{(x+y)^2} = 1$

6. $\frac{5x^2-5x}{1-x} = \frac{5x(x-1)}{-1(x-1)} = -5x$

B. Multiplication and Division

1. $\frac{x}{a} \cdot \frac{a}{b} = \frac{x}{b}$

2. $\frac{4a^2c}{b^3} \cdot \frac{3b^2}{12ac^2} = \frac{a}{bc}$

3. $(x-5)\frac{(x+4)}{5x-25} = \frac{(x-5)}{1} \cdot \frac{(x+4)}{5(x-5)} = \frac{(x+4)}{5}$

4. $\frac{u^2-4}{3} \cdot \frac{4}{u-2} = \frac{(u-2)(u+2)4}{3(u-2)} = \frac{4(u+2)}{3}$

5. $\frac{x^2+5x+6}{x^2-2x-15} \cdot \frac{x^2-x-20}{x^2+x-2} = \frac{(x+2)(x+3)(x-5)(x+4)}{(x-5)(x+3)(x+2)(x-1)} = \frac{(x+4)}{(x-1)}$

6. $\frac{x^2+4x+4}{2x^2-8} \div \frac{x^2+2x}{4x-8} = \frac{x^2+4x+4}{2x^2-8} \cdot \frac{4x-8}{x^2+2x} = \frac{(x+2)(x+2)4(x-2)}{2(x-2)(x+2)x(x+2)} = \frac{2}{x}$

7. $\frac{x^2+4x}{x^2-16} \div \frac{x^2+8x+15}{x^2+x-20} = \frac{x^2+4x}{x^2-16} \cdot \frac{x^2+x-20}{x^2+8x+15} = \frac{x(x+4)(x+5)(x-4)}{(x+4)(x-4)(x+5)(x+3)} = \frac{x}{(x+3)}$

8. $\frac{t^2-4}{t^2-25} \div \frac{t-2}{t-5} = \frac{t^2-4}{t^2-25} \cdot \frac{t-5}{t-2} = \frac{(t-2)(t+2)(t-5)}{(t-5)(t+5)(t-2)} = \frac{(t+2)}{(t+5)}$

9. $\frac{y}{y-5} \cdot \frac{y^2-6y+5}{y^2-1} \cdot \frac{y^2-4y-5}{y^2-5y} = \frac{y(y-5)(y-1)(y-5)(y+1)}{(y-5)(y-1)(y+1)y(y-5)} = 1$

C. Addition and Subtraction

1. $\frac{5y}{x^3y} + \frac{7y}{x^3y} = \frac{5y+7y}{x^3y} = \frac{12y}{x^3y} = \frac{12}{x^3}$

2. $\dfrac{3x^2+5x-2}{x+2}-\dfrac{x^2-6x+3}{x+2}=\dfrac{\left(3x^2+5x-2\right)-\left(x^2-6x+3\right)}{(x+2)}=$
$\dfrac{3x^2+5x-2-x^2+6x-3}{(x+2)}=\dfrac{2x^2+11x-5}{(x+2)}$

3. $\dfrac{6}{x^2-4}-\dfrac{4}{x-2}=\dfrac{6-4(x+2)}{(x-2)(x+2)}=\dfrac{6-4x-8}{(x-2)(x+2)}=\dfrac{-4x-2}{(x-2)(x+2)}=\dfrac{-2(2x+1)}{(x-2)(x+2)}$
$(x-2)(x+2)$ (x - 2)

4. $\dfrac{-8}{x^2-x-12}+\dfrac{3}{x^2-16}=\dfrac{-8(x+4)+3(x+3)}{(x-4)(x+3)(x+4)}=\dfrac{-8x-32+3x+9}{(x-4)(x+3)(x+4)}=\dfrac{(-5x-23)}{(x-4)(x+3)(x+4)}$
$(x-4)(x+3)$ (x - 4)(x + 4)

5. $\dfrac{-4}{2x^2+5x+3}+\dfrac{2}{4x^2-9}=$ $\dfrac{-4(2x-3)+2(x+1)}{(2x+3)(x+1)(2x-3)}=\dfrac{-8x+12+2x+2}{(2x+3)(x+1)(2x-3)}=$
$(2x+3)(x+1)$ (2x - 3)(2x + 3) $\dfrac{-6x+14}{(2x+3)(x+1)(2x-3)}=\dfrac{-2(3x+7)}{(2x+3)(x+1)(2x-3)}$

D. Solving Equations

1. $\dfrac{2}{5}+\dfrac{t}{4}=1\rightarrow(5)(4)\dfrac{2}{5}+(5)(4)\dfrac{t}{4}=(5)(4)1\rightarrow 8+5t=20\rightarrow 5t=12\rightarrow t=\dfrac{12}{5}$

2. $\dfrac{x+1}{3}-\dfrac{x+2}{6}=\dfrac{x+5}{4}\rightarrow(3)(6)(4)\dfrac{(x+1)}{3}-(3)(6)(4)\dfrac{(x+2)}{6}=(3)(6)(4)\dfrac{(x+5)}{4}\rightarrow$
$24(x+1)-12(x+2)=18(x+5)\rightarrow 24x+24-12x-24=18x+90\rightarrow 12x=18x+90\rightarrow$
$-6x=90\rightarrow x=-15$

3. $\dfrac{x}{3}+\dfrac{x}{4}=\dfrac{7}{2}\rightarrow(3)(4)(2)\dfrac{x}{3}+(3)(4)(2)\dfrac{x}{4}=(3)(4)(2)\dfrac{7}{2}\rightarrow 8x+6x=84\rightarrow$
$14x=84\rightarrow x=6$

$$\frac{4}{x-5}+\frac{3}{x+5}=\frac{40}{x^2-25}$$
$$(x-5)\ \ (x+5)\ \ (x-5)(x+5)$$

4. $\rightarrow (x-5)(x+5)\dfrac{4}{(x-5)}+(x-5)(x+5)\dfrac{3}{(x+5)}=(x-5)(x+5)\dfrac{40}{(x-5)(x+5)}\rightarrow$

$4(x+5)+3(x-5)=40\rightarrow 4x+20+3x-15=40\rightarrow 7x+5=40\rightarrow$

$7x=35\rightarrow x=5$

$$\frac{5}{x-10}+\frac{2}{x-4}=\frac{9}{x^2-14x+40}$$
$$(x-10)\ (x-4)\ \ (x-10)(x-4)$$

5.

$\rightarrow (x-10)(x-4)\dfrac{5}{(x-10)}+(x-10)(x-4)\dfrac{2}{(x-4)}=(x-10)(x-4)\dfrac{9}{(x-10)(x-4)}\rightarrow$

$5(x-4)+2(x-10)=9\rightarrow 5x-20+2x-20=9\rightarrow 7x-40=9\rightarrow 7x=49\rightarrow x=7$

$\dfrac{5x}{x+1}+\dfrac{4}{x}=9\rightarrow x(x+1)\dfrac{5x}{(x+1)}+x(x+1)\dfrac{4}{x}=x(x+1)9\rightarrow$

6. $x\cdot 5x+4(x+1)=9x(x+1)\rightarrow 5x^2+4x+4=9x^2+9x\rightarrow$

$-4x^2-5x+4=0\rightarrow 4x^2+5x-4=0$ (can' t be solved beyond this point)