

## Quadratic and Linear Functions

1. Convert each of the following into the form  $y = a(x - h)^2 + k$

a)  $y = 3x^2 + 2x + 1$

$$\begin{aligned} y - 1 &= 3\left(x^2 + \frac{2}{3}x\right) \\ y - 1 + \frac{4}{12} &= 3\left(x^2 + \frac{2}{3}x + \frac{4}{36}\right) \\ y + \frac{(-1) \cdot 12 + 4}{12} &= 3\left(x + \frac{4}{6}\right)^2 \\ y - \frac{8}{12} &= 3\left(x + \frac{4}{6}\right)^2 \\ y &= 3\left(x + \frac{4}{6}\right)^2 + \frac{8}{12} \end{aligned}$$

b)  $y = -2x^2 + 2x - 1$

$$\begin{aligned} y + 1 &= -2\left(x^2 - \frac{2}{2}x\right) \\ y + 1 - \frac{4}{8} &= -2\left(x^2 - \frac{2}{2}x + \frac{4}{16}\right) \\ y + \frac{(1) \cdot 8 - 4}{8} &= -2\left(x - \frac{2}{4}\right)^2 \\ y + \frac{4}{8} &= -2\left(x - \frac{2}{4}\right)^2 \\ y &= -2\left(x - \frac{2}{4}\right)^2 - \frac{4}{8} \end{aligned}$$

2. For each of the given questions determine the indicated information:

a)  $y = -3(x + 2)^2 + 1$

1. direction of opening      down
2. shape      narrow
3. max/min point      max
4. max/min value       $y = 1$
5. axis of symmetry       $x = -2$
6. coordinates of vertex       $(-2, 1)$
7. y-intercept      1
8. domain of function       $x \in R$
9. range of function       $y \leq 1$

10. Table of Values

|   |     |     |    |     |     |
|---|-----|-----|----|-----|-----|
| x | 5   | -4  | -2 | 0   | 1   |
| y | -26 | -11 | 1  | -11 | -26 |

b)  $y = 4(x - 3)^2 - 2$

1. Direction of opening      up
2. Shape      narrow
3. max/min point      min
4. Max/min value       $y = -2$
5. Axis of symmetry       $x = 3$
6. Coordinates of vertex       $(3, 2)$
7. y-intercept      -2
8. Domain of function       $x \in R$
9. Range of function       $y \geq -2$

10. Table of Values

|   |    |    |   |    |    |
|---|----|----|---|----|----|
| x | -1 | 0  | 3 | 6  | 7  |
| y | 62 | 34 | 2 | 34 | 62 |

3. Using the appropriate formula determine the equation of the axis of symmetry, the min/max value and the coordinated of the vertex of the equation  $y = 2x^2 - 6x - 5$

$$x = -\frac{b}{2a}$$

$$x = -\frac{(-6)}{2(2)}$$

$$x = \frac{6}{4}$$

axis of symmetry

$$y = \frac{4ac - b^2}{4a}$$

$$y = \frac{4(2)(-5) - (-6)^2}{4(2)}$$

$$y = \frac{-40 - 36}{8} = -\frac{76}{8}$$

min/max value

$$\left(\frac{6}{4}, -\frac{76}{8}\right)$$

vertex

4. Determine the required info:

a) slope and y-intercept of the equation:  $5x - 8y = -12$

$$-8y = -5x - 12$$

$$y = \frac{-5}{-8}x - \frac{12}{-8}$$

$$m = \frac{5}{8}, b = \frac{12}{8}$$

b) slope, midpoint, distance of the line joining the points  $(-5, -9)$  and  $(-3, 11)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-9 - 11}{-5 - (-3)}$$

$$m = \frac{-20}{-2} = \frac{20}{2} = 10$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M\left(\frac{-5 + (-3)}{2}, \frac{-9 + 11}{2}\right)$$

$$M\left(\frac{-8}{2}, \frac{2}{2}\right) = M(-4, 1)$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{((-5 - (-3))^2 + (-9 - 11)^2)}$$

$$d = \sqrt{(-2)^2 + (-20)^2} = \sqrt{404}$$

5. Determine the equation of the line given: **(NO FRACTIONAL COEFFICIENTS)**

a)  $m = 2/3$  and  $b = -6$

$$y = mx + b \Rightarrow y = \frac{2}{3}x + (-6) \Rightarrow 3(y) = 3 \cdot \frac{2}{3}x + (-6) \cdot 3 \Rightarrow 3y = 2x - 18$$

b)  $m = -4$  and contains the point  $(-2, -4)$

$$(y_2 - y_1) = m(x_2 - x_1)$$

$$(y - (-4)) = -4(x - (-2))$$

$$y + 4 = -4x - 8$$

$$y = -4x - 12$$

c) passes through the points  $(2, 7)$  and  $(-4, -3)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - (-3)}{2 - (-4)} = \frac{10}{6} = \frac{5}{3}$$

$$(y_2 - y_1) = m(x_2 - x_1) \Rightarrow (y - 7) = \frac{5}{3}(x - 2) \Rightarrow 3 \cdot (y - 7) = 3 \cdot \frac{5}{3}(x - 2) \Rightarrow 3y - 21 = 5x - 10$$

$$3y = 5x + 11$$

d) passes through the point (1,3) and is parallel to a line with equation  $5x - 3y = 4$

$$-3y = -5x + 4 \Rightarrow y = \frac{-5}{-3}x + \frac{4}{-3} \Rightarrow m_1 = \frac{5}{3}, \Rightarrow m_2 = \frac{5}{3}$$

$$(y_2 - y_1) = m(x_2 - x_1) \Rightarrow (y - 3) = \frac{5}{3}(x - 1) \Rightarrow 3 \cdot (y - 3) = 3 \cdot \frac{5}{3}(x - 1) \Rightarrow$$

$$3y - 3 = 5x - 5 \Rightarrow 3y = 5x - 2$$

e) of the perpendicular bisector of the line segment joining the points (6, 2) and (10, -4)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-4)}{6 - 10} = \frac{6}{-4} = -\frac{6}{4}$$

$$m_1 = -\frac{6}{4}, \therefore m_2 = \frac{4}{6}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{6 + 10}{2}, \frac{2 + (-4)}{2}\right) = \left(\frac{16}{2}, \frac{-2}{2}\right) = (8, -1)$$

$$(y_2 - y_1) = m(x_2 - x_1) \Rightarrow (y - (-1)) = \frac{4}{6}(x - 8) \Rightarrow 6 \cdot (y - (-1)) = 6 \cdot \frac{4}{6}(x - 8) \Rightarrow$$

$$6y + 6 = 4x - 32 \Rightarrow 6y = 4x - 38$$