Factoring

Grouping:

1. $b^3 - b^2 + bc + bd = b(b^2 - b + c + d)$

2. $xy + xz + wy + wz = (xy + xz) + (wy + wz) = x(y + z) + w(y + z) = (y + z)(x + w)$

3. $y^2 - 8y - y + 8 = (y^2 - 8y) + (-y + 8) = y(y - 8) - 1(y - 8) = (y - 8)(y - 1)$

4. $x^3 - x^2 + 3x - 3 = (x^3 - x^2) + (3x - 3) = x^2(x - 1) + 3(x - 1) = (x - 1)(x^2 + 3)$

5. $z^2 - 8z - z + 8 = (z^2 - 8z) + (-z + 8) = z(z - 8) - 1(z - 8) = (z - 8)(z - 1)$

6. $a^2 + 6a - 2a - 12 = (a^2 + 6a) + (-2a - 12) = a(a + 6) - 2(a + 6) = (a + 6)(a - 2)$

7. $2x^4 + 6x^2 + 5x^2 + 15 = (2x^4 + 6x^2) + (5x^2 + 15) = 2x^2(x^2 + 3) + 5(x^2 + 3) = (x^2 + 3)(2x^2 + 5)$

8. $2xy - x^2y - 6 + 3x = (2xy - x^2y) + (-6 + 3x) = xy(2 - x) - 3(2 - x) = (2 - x)(xy - 3)$

9. $x^2 + 2xy + y^2 - 25 = (x^2 + 2xy + y^2) - 25 = (x + y)^2 - 25 = [(x + y) + 5][(x + y) - 5]$

10. $x^2 - 2xy + y^2 - 49 = (x^2 - 2xy + y^2) - 49 = (x - y)^2 - 49 = [(x - y) + 9][(x - y) - 9]$

11. $a^2 - 2a + 1 - 4b^2 = [a^2 - 2a + 1] - 4b^2 = (a - 1)^2 - 4b^2 = [(a - 1) + 2b][(a - 1) - 2b]$

12. $x^2 + 4xy + 4y^2 - 9z^2 = (x^2 + 4xy + 4y^2) - 9z^2 = (x + 2y)^2 - 9z^2 = [(x + 2y) + 3z][(x + 2y) - 3z]$

13.
$2a^2 + 4ab + 2b^2 - 50d^2 = 2(a^2 + 2ab + b^2 - 25d^2) = 2([a^2 + 2ab + b^2] - 25d^2) =$
$2[(a + b)^2 - 25d^2] = 2[(a + b) + 5d][(a + b) - 5d]$

14.
$12t^2 + 12t + 3 - 3v^2 = 3(4t^2 + 4t + 1 - v^2) = 3([4t^2 + 4t + 1] - v^2) =$
$3[(2t + 1)^2 - v^2] = 3[(2t + 1) + v][(2t + 1) - v]$

15. $6x + 6y + ax + ay = (6x + 6y) + (ax + ay) = 6(x + y) + a(x + y) = (x + y)(6 + a)$

16. $3a - 3b + ca - cb = (3a - 3b) + (ca - cb) = 3(a - b) + c(a - b) = (a - b)(3 + c)$

17. $x^2 + 6x + 9 - z^2 = (x^2 + 6x + 9) - z^2 = (x + 3)^2 - z^2 = [(x + 3) + z][(x + 3) - z]$

18. $y^2 - 10x + 25 - x^2 = (y^2 - 10x + 25) - x^2 = (y - 5)^2 - x^2 = [(y - 5) + x][(y - 5) - x]$

19. $ax^2 + bx^2 - 9a - 9b = (ax^2 + bx^2) + (-9a - 9b) = x^2(a + b) - 9(a + b) =$
$(a + b)(x^2 - 9) = (a + b)(x + 3)(x - 3)$

20. $xy^2 - y^3 - 4x + 4y = (xy^2 - y^3) + (-4x + 4y) = y^2(x - y) - 4(x - y) =$
$(x - y)(y^2 - 4) = (x - y)(x + 2)(x - 2)$

21. $3x + 5ax + 3y + 5ay = (3x + 5ax) + (3y + 5ay) = x(3 + 5a) + y(3 + 5a) = (3 + 5a)(x + y)$

22. $ax + 3by + 2a^2x + 6aby = (ax + 3by) + (2a^2x + 6aby) = (ax + 3by) + 2a(ax + 3by) =$
$(ax + 3by)(1 + 2a)$

23. $4ax - 5by - 8ax^2 + 10bxy = (4ax - 5by) + (-8ax^2 + 10bxy) = (4ax - 5by) - 2x(4ax - 5by) =$
$(4ax - 5by)(1 - 2x)$

24. $x^2 - 4y^2 - x + 2y = (x^2 - 4y^2) + (-x + 2y) = (x + 2y)(x - 2y) - 1(x - 2y) =$
$(x - 2y)[(x + 2y) - 1]$

25. $5x^2 - 3x + 3y - 5y^2 = (5x^2 - 5y^2) + (-3x + 3y) = 5(x + y)(x - y) - 3(x - y) =$
$(x - y)[5(x + y) - 3]$

26. $27x^3 - 12xy^2 + 6xy + 4y^2 = (27x^3 - 12xy^2) + (6xy + 4y^2) = 3x(9x^2 - 4y^2) + 2y(3x + 2y) =$
$3x(3x + 2y)(3x - 2y) + 2y(3x + 2y) = (3x + 2y)[3x(3x - 2y) + 2y]$

27. $4x^2 - 14x + 4xy - 7y + y^2 = (4x^2 + 4xy + y^2) + (-14x - 7y) = (2x + y)^2 - 7(2x + y) =$
$(2x + y)[(2x + y) - 7]$

28. $x^3 - x^2 + 2xy - y^2 - y^3 = (x^3 - y^3) + (-x^2 + 2xy - y^2) = (x^3 - y^3) - (x^2 - 2xy + y^2) =$
$(x - y)(x^2 + xy + y^2) - (x - y)(x - y) = (x - y)[(x^2 + xy + y^2) - (x - y)]$

$$x^3 + 3x^2 + 5x - 5y - 3xy - y^3 = \left(x^3 - y^3\right) + \left(3x^2 + 5x\right) + \left(-5y - 3xy\right) =$$

29. $(x - y)(x^2 + xy + y^2) + x(3x + 5) - y(3x + 5) = (x - y)(x^2 + xy + y^2) + (3x + 5)(x - y) =$

$(x - y)\left[(x^2 + xy + y^2) + (3x + 5)\right]$

30. $a^6 - a^3 + b^3 - b^6 = \left(a^6 - b^6\right) + \left(-a^3 + b^3\right) = (a^3 + b^3)(a^3 - b^3) - (a^3 - b^3) =$

$(a^3 - b^3)\left[(a^3 + b^3) - 1\right] = (a - b)(a^2 + ab + b^2)\left[(a + b)(a^2 - ab + b^2) - 1\right]$

31. $a^4 - 3a^2 + 3b^2 - b^4 = \left(a^4 - b^4\right) + \left(-3a^2 + 3b^2\right) = (a^2 + b^2)(a^2 - b^2) - 3(a^2 - b^2) =$

$(a^2 - b^2)\left[(a^2 + b^2) - 3\right] = (a + b)(a - b)\left[(a^2 + b^2) - 3\right]$

32. $3x^3 - 12xy^2 + 5xy - 10y^2 = \left(3x^3 - 12xy^2\right) + \left(5xy - 10y^2\right) = 3x(x^2 - 4y^2) + 5y(x - 2y) =$

$3x(x + 2y)(x - 2y) + 5y(x - 2y) = (x - 2y)\left[3x(x + 2y) + 5y\right]$

33. $3x - 9x^2 + 4y + 16y^2 = (3x + 4y) + \left(-9x^2 + 16y^2\right) = (3x + 4y) - \left(9x^2 - 16y^2\right) =$

$(3x + 4y) - (3x + 4y)(3x - 4y) = (3x + 4y)\left[1 - (3x - 4y)\right]$

34. $2x^2y - 7x^2z + 10xy - 35xz = \left(2x^2y - 7x^2z\right) + \left(10xy - 35xz\right) = x^2(2y - 7z) + 5x(2y - 7z) =$

$(2y - 7z)(x^2 + 5x) = (2y - 7z)(x)(x + 5)$