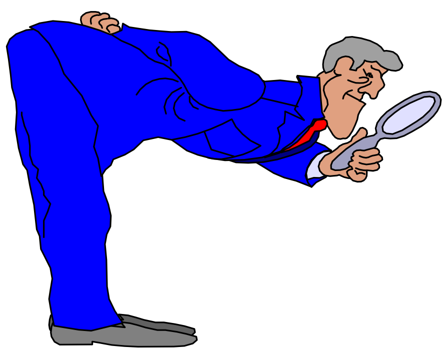
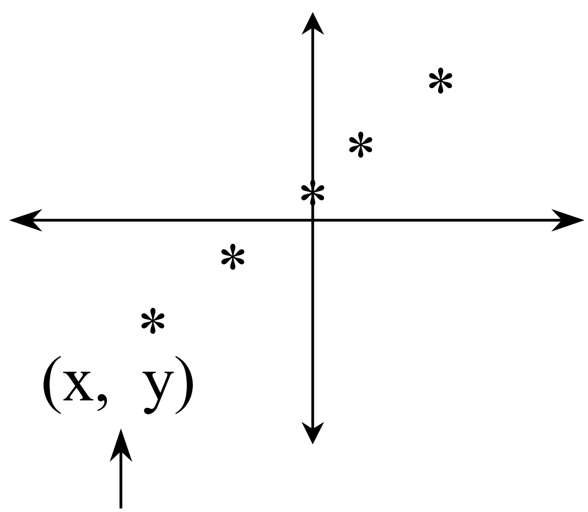
Linear Functions Two

Contents

Definition of a Linear Function

- a set of points that have a constant slope



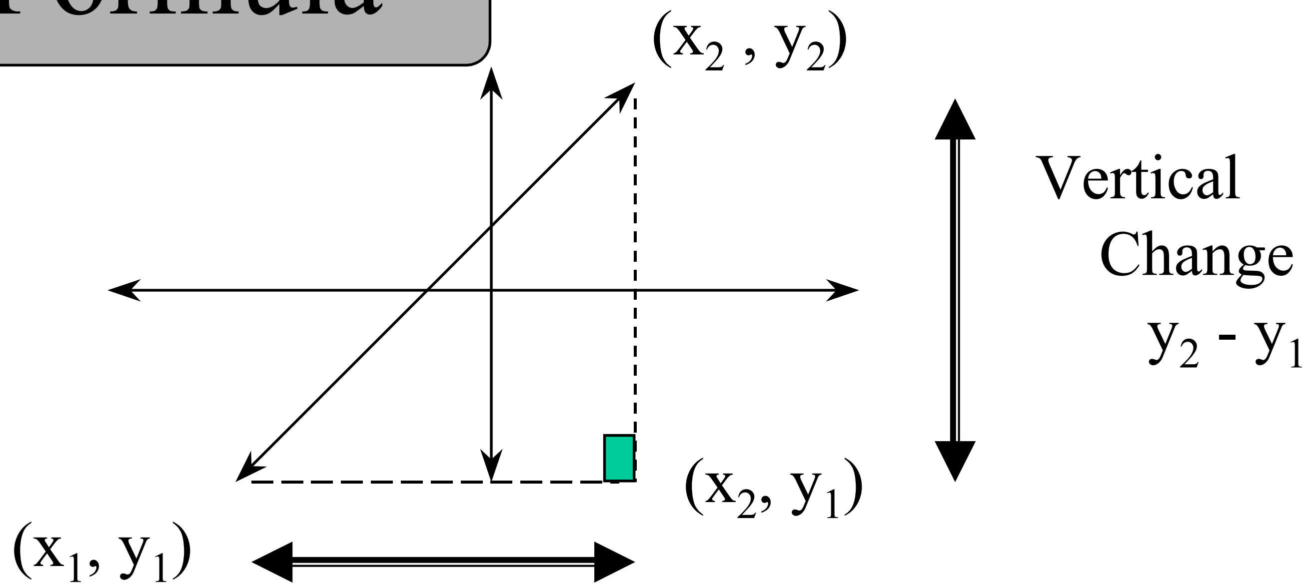


Remember each point on a coordinate plane has a x-coordinate and a y-coordinate

When graphed, lines can be pictured in one of four different scenarios.If graphs are read from left to right. some lines slope upward and others slope downward -- some are really steep, while others have a gentle rise or fall. Some of the lines are straight up and down--vertical, while others lie flat -- horizontal.

The slope of a line has been defined as the change in y over the change in x, or the rise over the run. This can be explained with a formula: $m = (y_2 - y_1)/(x_2 - x_1)$. To find the slope, you pick any two points on the line and find the change in y, and then divide it by the change in x.

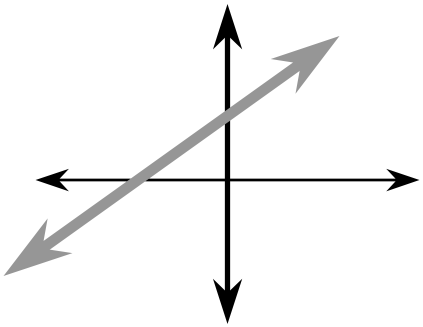
Slope Formula



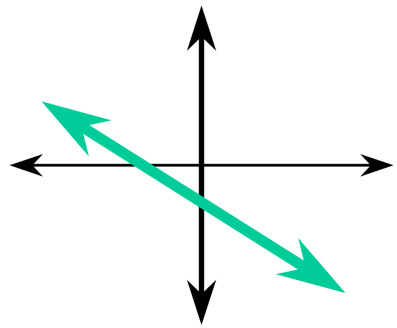
Horizontal Change $x_2 - x_1$

$$m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

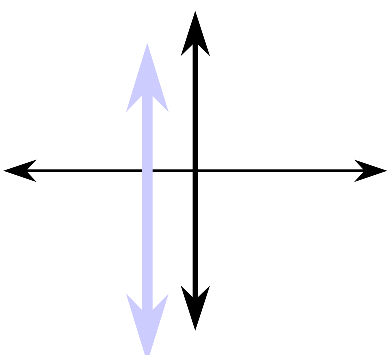
Types of Slopes



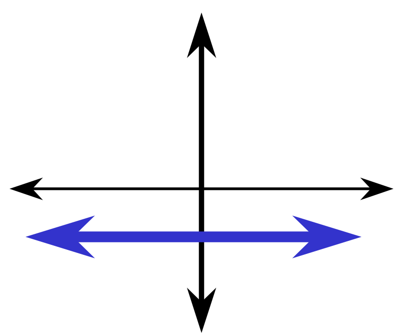
positive slope
line moves up from left to right



negative slope
line moves down from left to right



no slope
a vertical line



zero slope
a horizontal line

Application of The Slope Formula:

Example 1:

Determine the slope of the line joining the points (7, 5) and (-3, -3)

Hints: 1. It is probably easiest to use the first ordered pair for the subscript 2. ($x_2 = 7$, $y_2 = 5$, $x_1 = -3$, $y_1 = -3$)

2. Remember that the negative (-)sign that appears in the formula is part of the formula.
3. When substituting negatives values into the formula place in ( ) , then simplify the expression by removing the ( ) before doing any addition or subtraction.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-3)}{7-(-3)} = \frac{5+3}{7+3} = \frac{8}{10} = \frac{4}{5}$$

Example 2:

Determine the slope of the line given the equation of the line
Given $3x + 5y = 15$
Hints: 1. Determine two points that lie on the line (strongly recommend that you use "x" and "y" intercepts)
2. Once the points have been determined follow procedure from previous example

For the given equation:

1. x-intercept $3x + 5(0) = 15$, $x = 5$; y-intercept $3(0) + 5y = 15$, $y = 3$
2. Calculated points are (5, 0) and (0, 3)
3. Slope =

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - (3)}{5 - (0)} = \frac{-3}{5}$$

Other procedures for calculating slope

Determine the slope of the line 4x + 3y = 12

a) Intercept Points: 4x + 3(0) = 12, x = 3; (3, 0)

4(0) + 3y = 12, y = 4; (0, 4)

b) Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - (4)}{3 - (0)} = \frac{-4}{3}$$

Solve the equation for one positive "y"

4x + 3y = 12

3y = -4x + 12

y = -4/3 x + 12/3 or y = -4/3 x + 4

Note: the coefficient of x -4/3 is the same as the slope we calculated in the above example (step b) and the value 4 is the same as the y-intercept that we calculated above (step a). If slope equals "m"and the y-intercept "b", we can re-write the equation in the form

y = mx + b -- the slope-intercept formula

The information the previous slide allows us to conclude that if given a linear equation we can determine the slope and the y-intercept by **solving for one positive "y".**

Examples:

Given: $7x - 3y = -12$

$-3y = -7x - 12$

$y = -7/-3\ x - 12/-3$

$m = 7/3$ and $b = 4$

Given: $Ax + By = C$

$By = -Ax + C$

$y = -A/B\ x + C/B$

$m = -A/B$ and $b = C/B$

If we compare the two examples above we can see that if we wished we could use a formula to find the slope and y-intercept.

Slope = m = -A/B and y-intercept = b = C/B

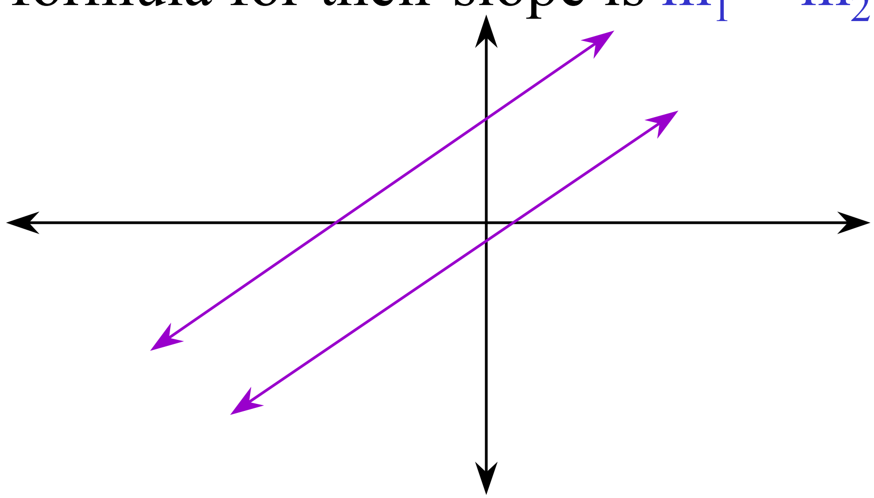
These two formula provide you with an alternative to solving for one positive 'y". Remember to pay attention to the negative that is part of the formula.

Other Special Lines

Parallel Lines (//)

Parallel lines are defined as two lines that have the same slope but have no points in common.

The formula for their slope is $m_1 = m_2$

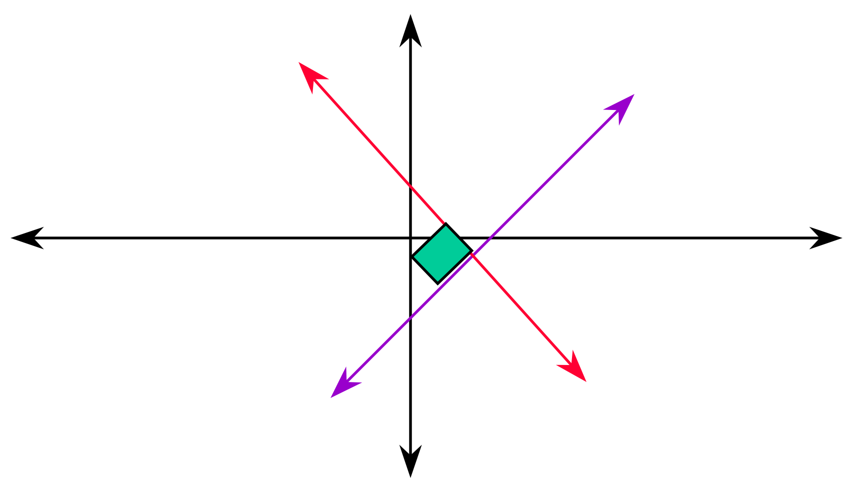


Perpendicular Lines ($\perp$)

Two lines that meet at right angles to one another and share one intersection point. Their slopes are negative reciprocals of one another.

The formula for their slope is $m_1 * m_2 = -1$

Example : if $m_1 = 2/3$ then $m_2 = -3/2$



Applications

1. To determine whether three points lines on the same line (this is asking the equation "Are the three points collinear?" -- points that are on the same line and having the same slope).
   Given: (-2, 9), (3, 4) and (5, 2)

(-2, 9) and (3, 4) $\quad m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-4}{5-3} = \dfrac{-2}{2} = -1$

(3, 4) and (5, 2) $\quad m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4-9}{3-(-2)} = \dfrac{-5}{5} = -1$

(-2, 9) and (5, 2) $\quad m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-9}{5-(-2)} = \dfrac{-7}{7} = -1$

Slopes are the same therefore we can conclude that the three points lie on the same line. (the points are collinear)

2. To determine the whether two lines are perpendicular, parallel or intersecting.Remember that parallel lines will have the same slope, intersecting lines will have different slopes and perpendicular lines will have different slopes that are negative reciprocals of one another.

A) What type of lines are represented by the given two equations

$3x + 5y = -3$ | $-5x + 3y = 11$

$m = -A/B = -3/5$ | $m = -A/B = -(-5)/3 = 5/3$

lines are perpendicular: slopes different and $m_1 * m_2 = -1$

B) What type of lines are represented by the two lines defined by the following sets of two points (4, 7) (-3, 6) and (-7, 3) (5, 1)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-6}{4-(-3)} = \frac{1}{7} \text{ and } \frac{3-1}{-7-5} = \frac{2}{-12} = \frac{1}{6}$$

lines are intersecting: slopes different and $m_1 * m_2 \neq -1$

3. To determine whether the three coordinates that are given represent the three vertices of a right triangle. Remember that two sides must meet at right angles (perpendicular to one another) to form a right triangle.

Given : A(1, 2), B(5, 7), C(9, 2)

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{2-7}{1-5}=\frac{-5}{-4}=1$$

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{7-2}{5-9}=\frac{5}{-5}=-1$$

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{2-2}{1-9}=\frac{0}{-8}=0$$

Line segment AB is perpendicular to line segment BC because their slopes are negative reciprocals of one another. We can conclude that these vertices forma right triangle.

Assignment:

1. Determine the slope of the line through he given points:
   a) (7, 6) , (-5, -3)
   b) (-4, 11), (-2, 9)
   c) (-9, -1), (-5, -4)
2. Determine the slope of the line whose equation is given:
   a) $4x - 5y = 13$
   b) $-7x + 2y = 12$
3. Do the following points lie on the same line? (Are the three points collinear?) (5, 7), (-3, 3) and (-7, -1)
4. A line with slope -2 passes through the points (9, 3p) and (5, 2p). Find the value of "p".
5. Do the vertices A(-5, 7) , B(-1, 1) and C(5, 5) represent a right triangle?
6. What type of lines do these equations represent?

$$5x - 6y = -12 \text{ and } -10x + 12y = 8$$

Answer key

1. a) $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, $m = \dfrac{6 - (-3)}{7 - (-5)}$, $m = \dfrac{9}{12} = \dfrac{3}{4}$

b) $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, $m = \dfrac{11 - 9}{(-4) - (-2)}$, $m = \dfrac{2}{-2} = -1$

c) $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, $m = \dfrac{(-1) - (-4)}{-9 - (-5)}$, $m = \dfrac{3}{-4} = -\dfrac{3}{4}$

2. a) y = -4/-5 x + 13/-5
m = 4/5

b) y = 7/2 x + 12/2
m = 7/2

3. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, $m = \dfrac{7 - 3}{5 - (-3)}$, $m = \dfrac{4}{8} = \dfrac{1}{2}$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$, $m = \dfrac{7 - (-1)}{5 - (-7)}$, $m = \dfrac{8}{12} = \dfrac{2}{3}$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$, $m = \dfrac{3 - (-1)}{-3 - (-7)}$, $m = \dfrac{4}{4} = 1$

Lines are not collinear because slopes are different

4. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$-2 = \dfrac{3p - 2p}{9 - 5}$

$-2 = \dfrac{p}{4} \Rightarrow p = -8$

Yes, because two slopes are negative reciprocals of one another. This indicates that two lines are perpendicular to one another, therefore a right triangle.

5.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $\qquad$ $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $\qquad$ $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$m = \dfrac{7 - 1}{-5 - (-1)}$ $\qquad$ $m = \dfrac{7 - 5}{-5 - 5}$ $\qquad$ $m = \dfrac{1 - 5}{-1 - 5}$

$m = \dfrac{6}{-4} = -\dfrac{3}{2}$ $\qquad$ $m = \dfrac{2}{-10} = -\dfrac{1}{5}$ $\qquad$ $m = \dfrac{-4}{-6} = \dfrac{2}{3}$

6. y = -5/-6 x - 12/-6 $\qquad$ y = 10/12 x + 8/12

m = 5/6, b = 2 $\qquad$ m = 5/6, b = 2/3

Since slopes are the same and the y-intercepts are different we can conclude that the lines are parallel

A special look at part of a line - **the line segment**

Definition : a set of points with a constant slope and two definite endpoints

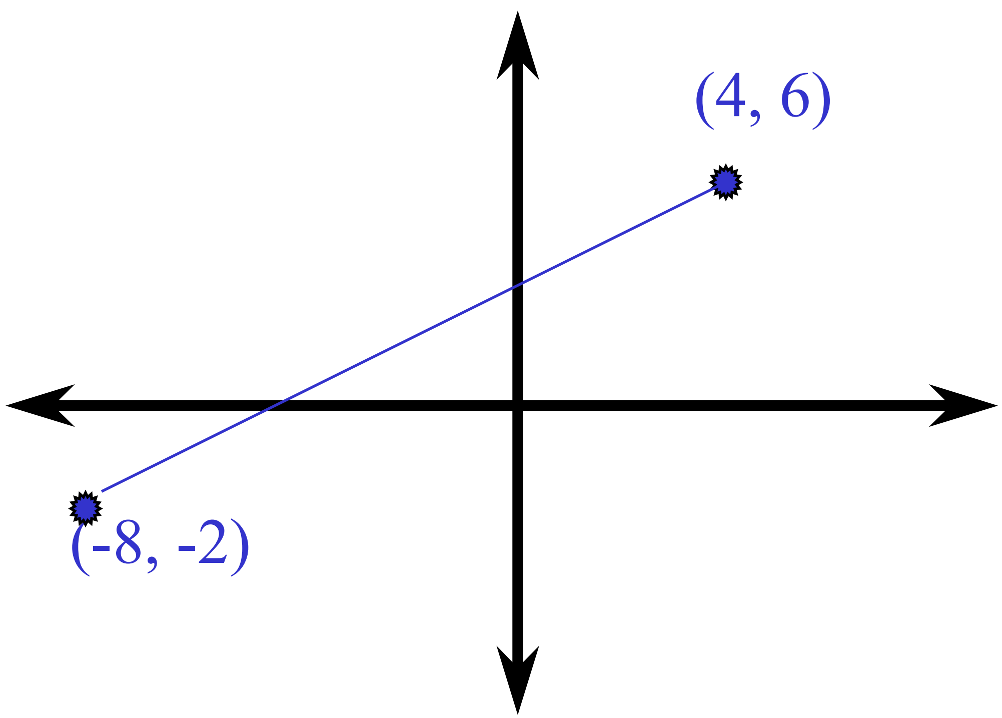
Types of calculations that can be made:

a) midpoint

b) distance

What is the midpoint of a line segment?

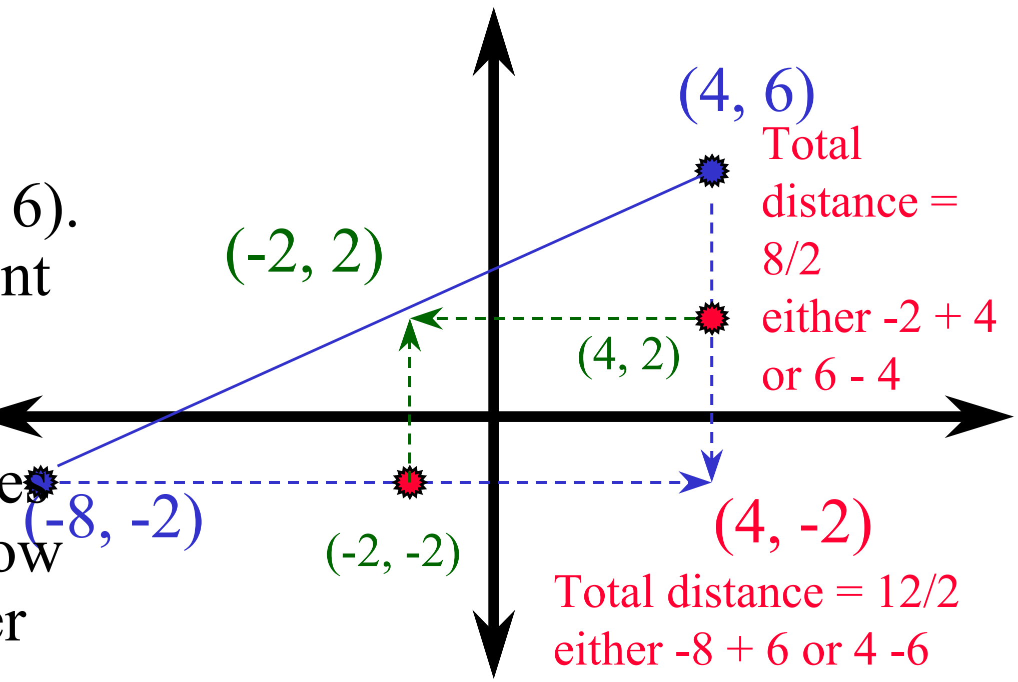
Let us consider the midpoint as the halfway point between two given points



How do we find the midpoint of the line segment defined by points (4, 6) and (-8, -2)

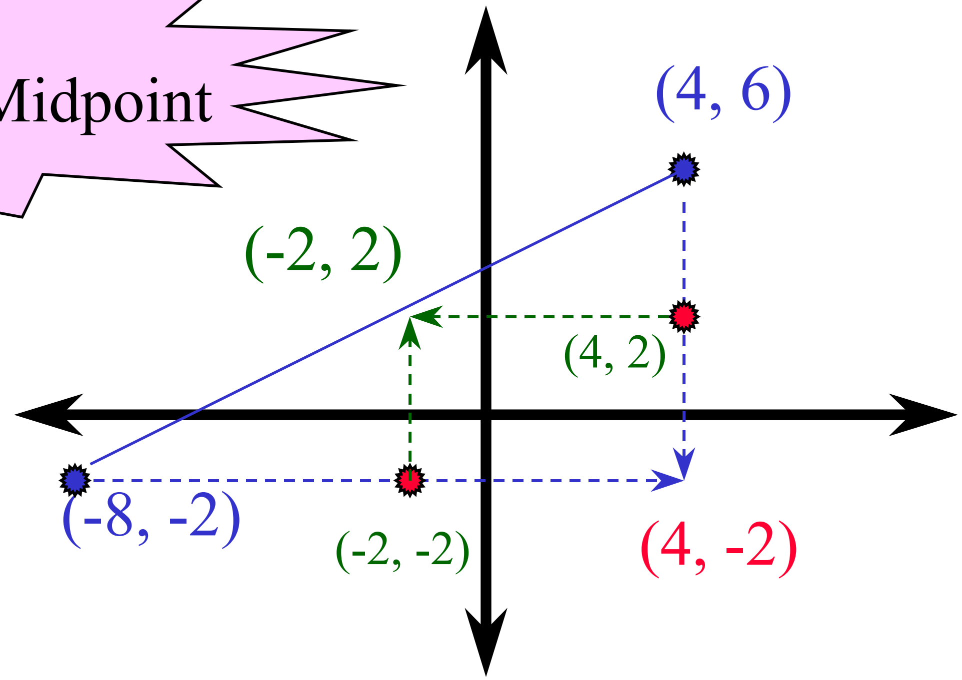
Note: when following the information on the next slide pay attention to how the midpoint is calculated as this will give you some clues as to the formula development

Step 1. draw a horizontal line from the point (-8, -2) and a vertical line from the point (4, 6). Determine the intersection point of these two lines (4, -2)
Step 2. determine the halfway point on the horizontal (changes in x-values) line by finding how far it is from -8 to 4 (remember distance is always positive)

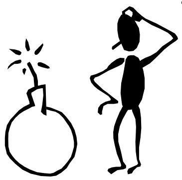


and dividing the result by two to find the halfway point. Add this result to the left most value of x or subtract from right most value.(red text)
Step 3. repeat this procedure for the vertical line (remember to add to the smaller value of y and subtract from the higher value)
Step 4. draw vertical and horizontal lines from these two points.
Step 5. the intersection point of these lines and the original line segment is the midpoint (-2, 2)

The Formula
for Calculation of the Midpoint



Instead of finding the distance between -8 and 4 and dividing this result by 2 and then adding or subtracting from one of the x's



one might realize that there is probably an easier method.

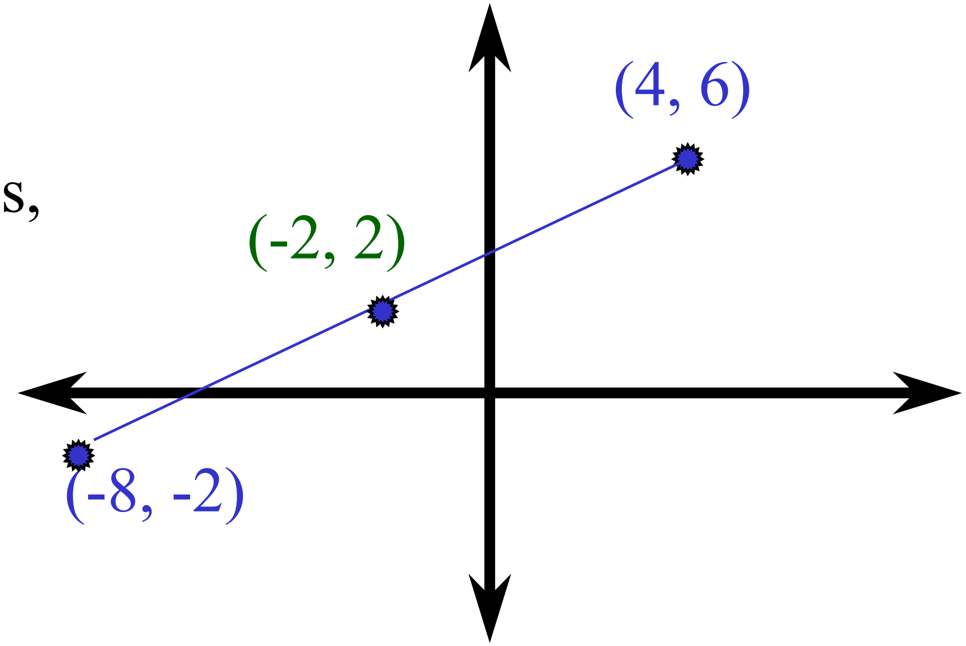
Let us try this:

Step 1: take the average of the two x-values $(-8 + 4)/2 = -2$

Step 2: take the average of the two y-values $(6 + -2)/2 = 2$

Step 3: Combine your answers into an ordered pair and you will note that your answer is the same as the midpoint we calculated using the previous method.

If I am taking the average of the x coordinates and the y coordinates, is it really necessary to draw the horizontal and vertical lines and find intersection points.



To reduce the need for diagrams and additional lines our best substitute is to use a formula and the formula is based on the idea of taking averages. The capital **M** will be the symbol used to indicate the midpoint formula.

$$M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Note: either point may be used for subscript "1" or "2" but be careful not to mix the values and it is recommended to use ( ) for each substitution prior to simplifying

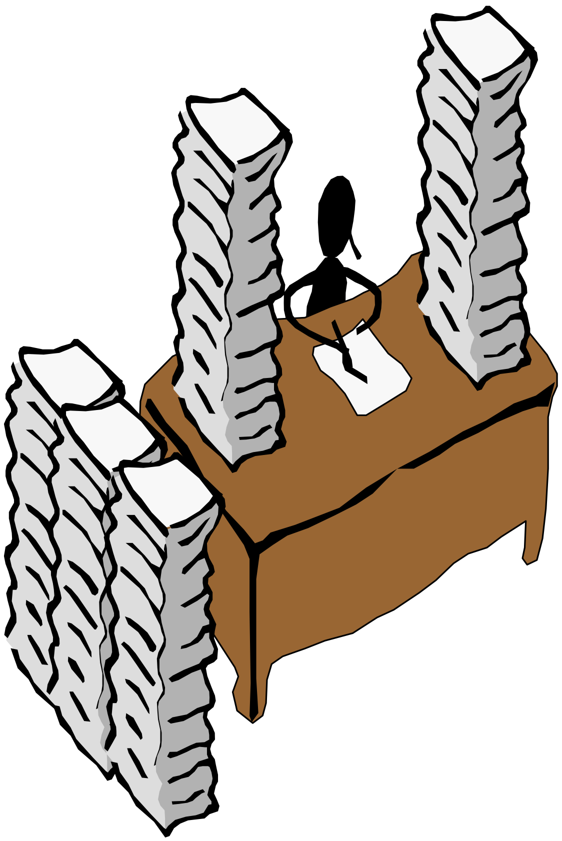
Application of the Midpoint Formula:

1. Using information from the previous example let us attempt to find the midpoint of the line segment defined by the endpoints (4, 6) and (-8, -2).

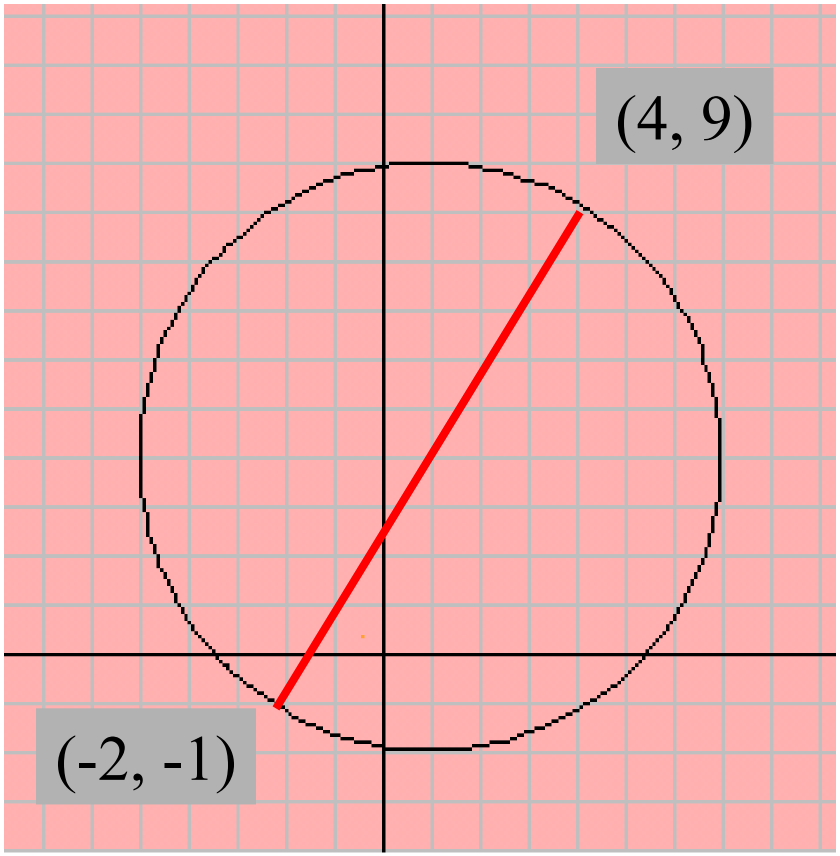
$$M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$M\left(\frac{4+(-8)}{2},\frac{6+(-2)}{2}\right)$$

$$M\left(\frac{-4}{2},\frac{4}{2}\right)=(-2,2)$$



2. The coordinates of the center of a circle knowing the endpoints of the diameter



$$M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$M\left(\frac{4+(-2)}{2},\frac{9+(-1)}{2}\right)$$

$$M\left(\frac{2}{2},\frac{8}{2}\right)=(1,4)$$

Coordinates of the center of the circle (1, 4)

Finding the Distance Between Two Points
or finding the length of a line segment

Note: distance is always considered a positive value.

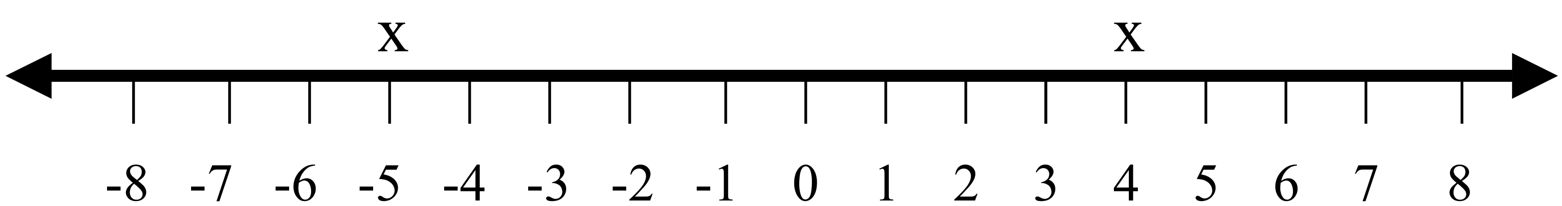
Two scenarios

a) using a number line (either vertical or horizontal)

b) using two points on a coordinate a plane

Distance on a Number Line

Find the distance (remember distance is always positive) between the indicated values.



Methods:

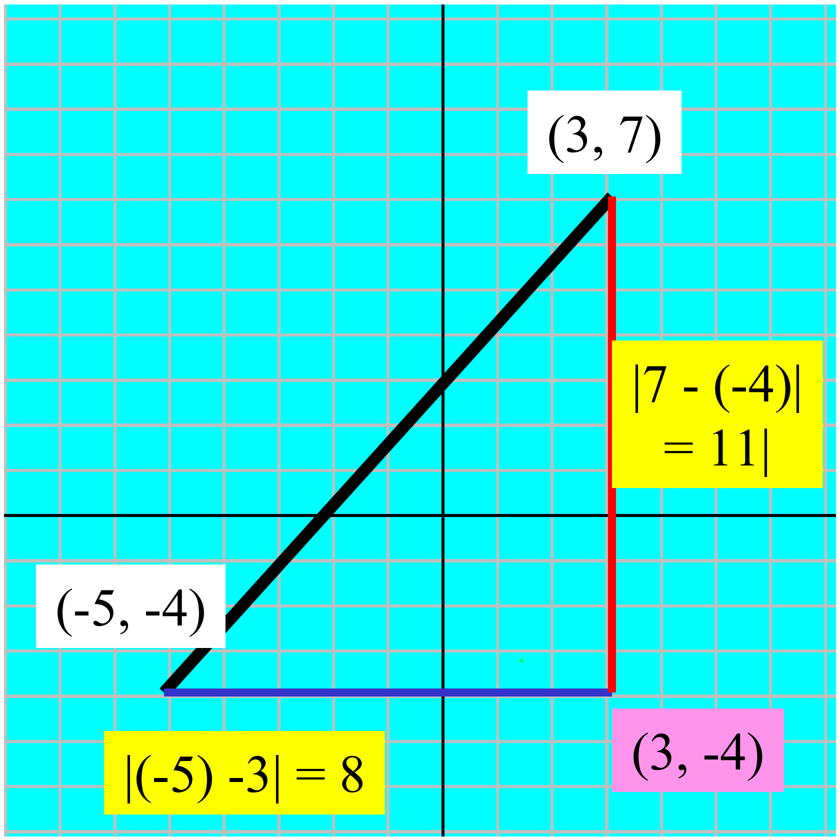
1. Count the number of units as you move from -5 to 4: distance = 9
2. Subtract one value from the next. Since distance must be positive we want to assure a positive answer so we take the absolute value of our answer: $|(-5) - 4| = |4 - (-5)| = 9$

The second method provides use with the opportunity to develop a formula that will work on a horizontal number line (or line parallel to it) or on a vertical number line (or line parallel to it). For:

a) horizontal lines (x-values) = $|x_2 - x_1|$

b) vertical lines (y-values) = $|y_2 - y_1|$

Distance between two points on a coordinate plane



Find the distance between the points (-5, -4) and (3, 7)

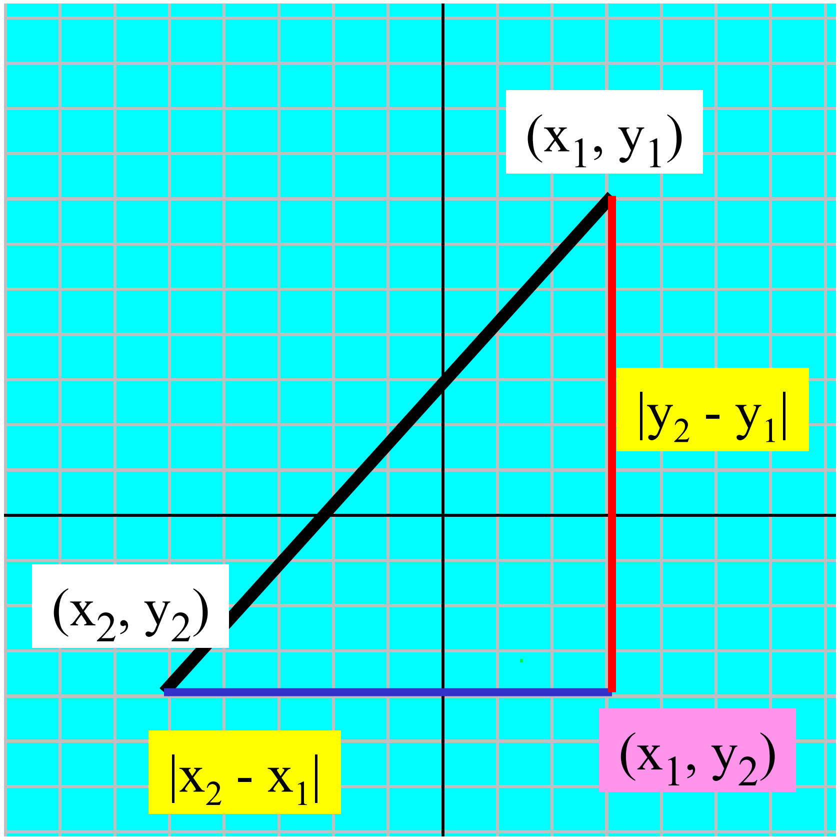
Steps:

1. Draw a horizontal line from the point (-5, -4) until it intersects a vertical line draw from the point (3, 7).
2. Determine the coordinates of this intersection point (3, -4)
3. Determine the distance between (-5, -4) and (3, -4) and between (3, 7) and (3, -4).
4. Use the right triangle theorem to determine the distance between (-5, -4) and (3, 7). $c^2 = a^2 + b^2$

$$c^2 = 8^2 + 11^2$$

$$c^2 = 185 \longrightarrow c = \sqrt{185} \text{ or } 13.6$$

Distance Formula



Find the distance between the points (x_1, y_1) and (x_2, y_2)

Steps:

1. Draw a horizontal line from the point (x_2, y_2) until it intersects a vertical line draw from the point (x_1, y_1) .
2. Determine the coordinates of this intersection point (x_1, y_2)
3. Determine the distance between (x_2, y_2) and (x_1, y_2) and between (x_1, y_1) and (x_1, y_2)
4. Use the right triangle theorem to determine the distance between (x_1, y_1) and (x_2, y_2). $c^2 = a^2 + b^2$ Let $d = c$ (d for distance)

$$d^2 = (|x_2 - x_1|)^2 + (|y_2 - y_1|)^2$$

$$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

Squaring removes the absolute symbol

Application of the distance formula

1. Determine the length of the line segment joining the points with the given coordinates (-7, -12) and (8, 14).

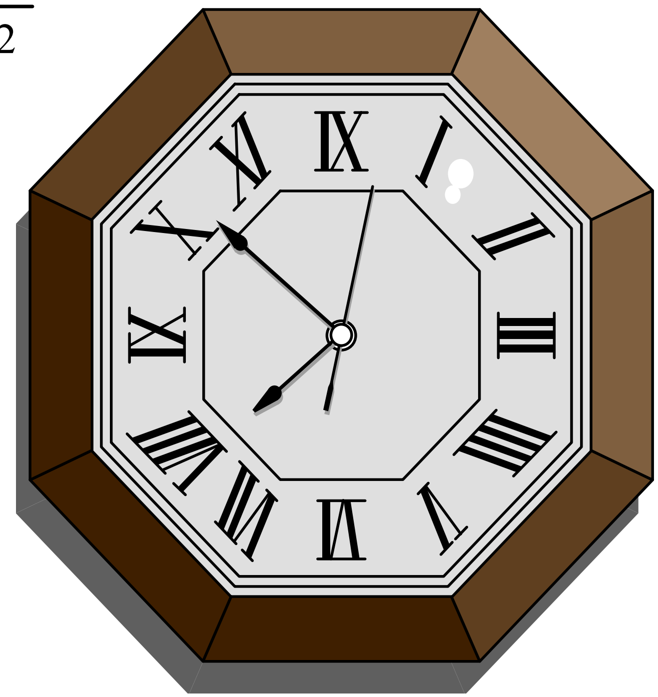
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{((-7) - 8)^2 + ((-12) - 14)^2}$$

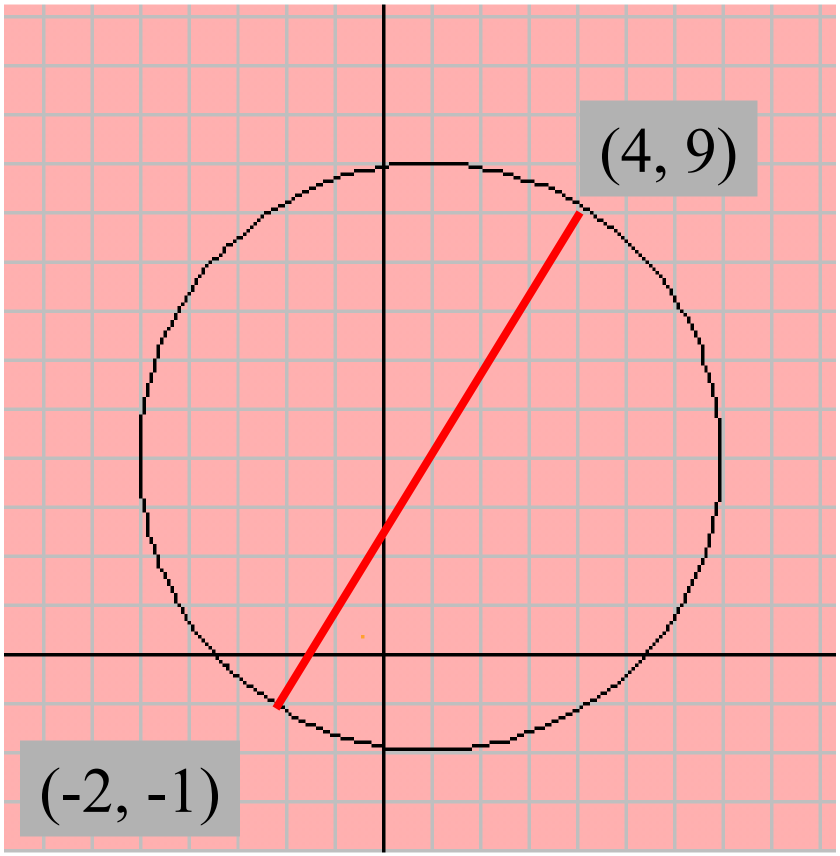
$$d = \sqrt{(-15)^2 + (-26)^2}$$

$$d = \sqrt{225 + 676}$$

$$d = \sqrt{901}$$



2. The radius of a circle knowing the endpoints of the diameter



$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d=\sqrt{(4-(-2))^2+(9-(-1))^2}$$

$$d=\sqrt{6^2+10^2}$$

$$d=\sqrt{36+100}$$

$$d=\sqrt{136}=2\sqrt{34}=11.66$$

The length of the diameter is 11.66 units therefore the length of the radius is one half the that length or 5.83 units
Remember: diameter = 2 times the radius

3. Determine the type of triangle (right, isosceles, equilateral,or scalene) and the perimeter of the triangle if given the three vertices at coordinates (-2, 6) (6, 8) and (2, -5) which define the given triangle.

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$d=\sqrt{((-2)-6)^2+(6-8)^2}$$
$$d=\sqrt{(-8)^2+(-2)^2}$$
$$d=\sqrt{64+4}$$
$$d=\sqrt{68}$$

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$d=\sqrt{((-2)-2)^2+(6-(-5))^2}$$
$$d=\sqrt{(-4)^2+(11)^2}$$
$$d=\sqrt{16+121}$$
$$d=\sqrt{137}$$

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$d=\sqrt{(6-2)^2+(8-(-5))^2}$$
$$d=\sqrt{(4)^2+(13)^2}$$
$$d=\sqrt{16+169}$$
$$d=\sqrt{185}$$

Type of triangle

-since no two or three sides are equal it is not isosceles or equilateral

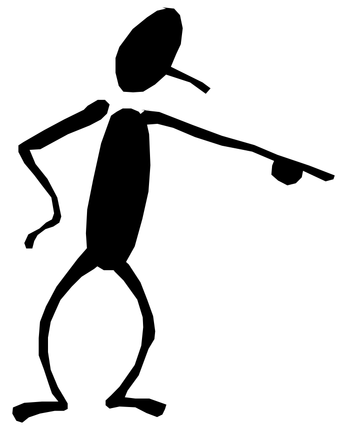
- is it scalene or a right scalene

$$c^2=a^2+b^2\Rightarrow$$
$$(\sqrt{182})^2=(\sqrt{68})^2+(\sqrt{137})^2\Rightarrow$$
$$182\neq 68+137 \text{ (Scalene)}$$

Perimeter = a + b + c

$$=\sqrt{182}+\sqrt{68}+\sqrt{137}$$

Assignment



1. Determine the slope of the line:
   - a) parallel to a line with slope $m_1 = -6/7$
   - b) perpendicular to a line with slope $m_1 = 5/3$
2. Determine the slope of the line parallel to and perpendicular to:
   - a) a line containing points (-3, 6) and (-5,-4)
   - b) a line with an equation of $5x - 3y = 6$
3. Determine the midpoint of and the distance between the given points (-4, 7) and (-1, -6)
4. Determine the center and the length of the radius of a circle that has endpoints of the diameter at (-5, 3) and (7, -11).
5. Determine whether the following coordinates represent the vertices of a right, scalene, equilateral, isosceles or right scalene triangle.

   (-5, -2), (-1, 6) and (7, 2)

Answer key:

1. a) $m_2 = -6/7$ b) $m_2 = -3/5$

2. a)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{6-(-4)}{-3-(-5)}$$

$$m = \frac{10}{2} = 5$$

parallel line slope = 5

perpendicular line slope = -1/5

b) y = -5/-3 x + 6/-3

$m_1 = 5/3$

parallel line slope $m_2 = 5/3$

perpendicular line slope $m_2 = -3/5$

3.

$$M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$M\left(\frac{-4+(-1)}{2},\frac{7+(-6)}{2}\right)$$

$$M\left(\frac{-5}{2},\frac{1}{2}\right)$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{((-4)-(-1))^2 + (7-(-6))^2}$$

$$d = \sqrt{(-3)^2 + (13)^2}$$

$$d = \sqrt{9+169}$$

$$d = \sqrt{178}$$

4. Center

$$M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$M\left(\frac{(-5)+7}{2},\frac{3+(-11)}{2}\right)$$

$$M\left(\frac{2}{2},\frac{-8}{2}\right)=(1,-4)$$

Radius

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d=\sqrt{((-5)-7)^2+(3-(-11))^2}$$

$$d=\sqrt{(-12)^2+(14)^2}$$

$$d=\sqrt{144+196}$$

$$d=\sqrt{340}=2\sqrt{85}$$

5.

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d=\sqrt{((-5)-(-1))^2+(-2-6)^2}$$

$$d=\sqrt{(-4)^2+(-8)^2}$$

$$d=\sqrt{16+64}$$

$$d=\sqrt{80}=4\sqrt{5}$$

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d=\sqrt{((-5)-7)^2+(-2-2)^2}$$

$$d=\sqrt{(-12)^2+(-4)^2}$$

$$d=\sqrt{144+16}$$

$$d=\sqrt{160}=4\sqrt{10}$$

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d=\sqrt{((-1)-7)^2+(6-2)^2}$$

$$d=\sqrt{(-8)^2+(4)^2}$$

$$d=\sqrt{64+16}$$

$$d=\sqrt{80}=4\sqrt{5}$$

Since two measurements the same the triangle is isosceles

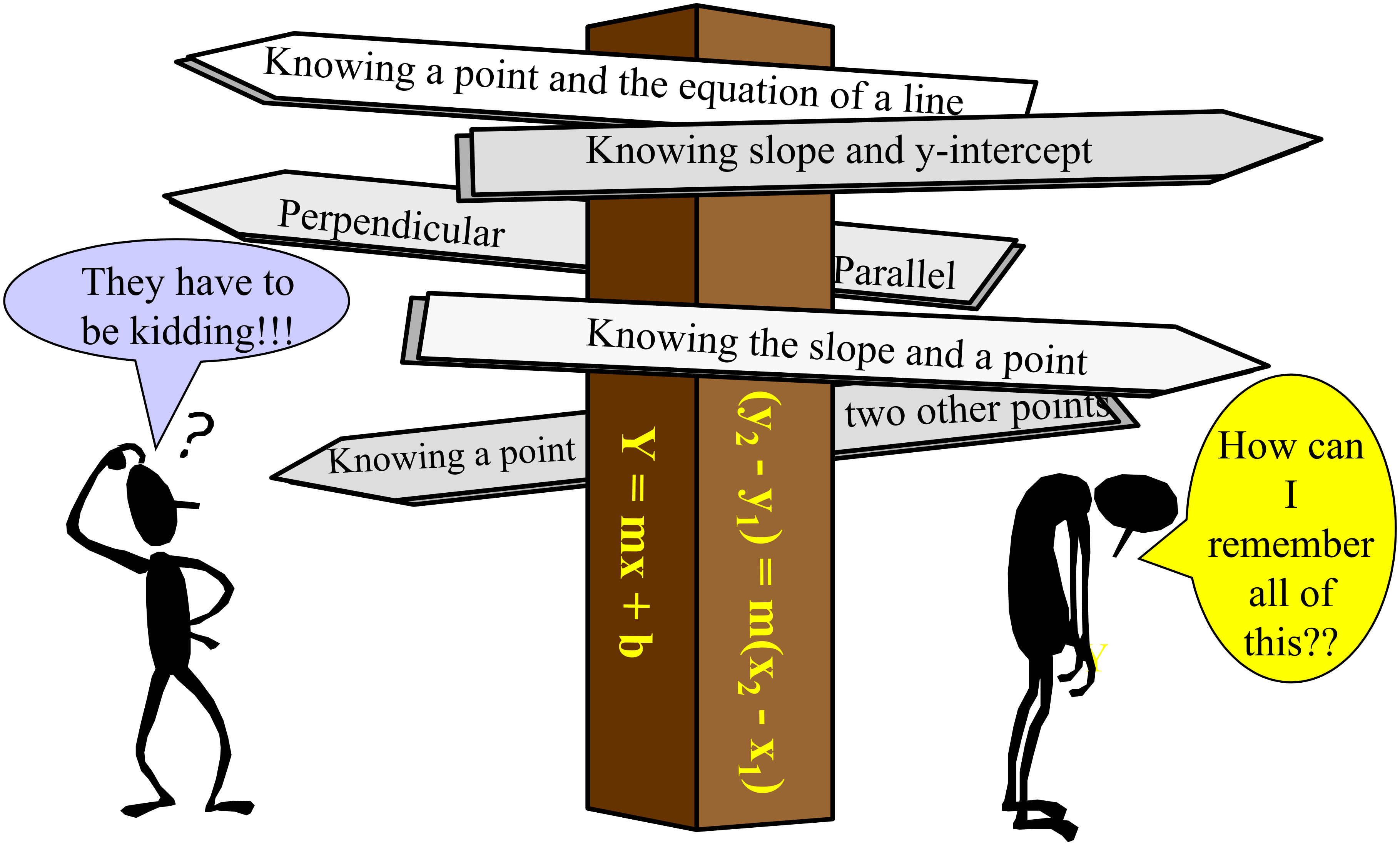
$$c^2=a^2+b^2\Rightarrow$$

$$(\sqrt{160})^2=(\sqrt{80})^2+(\sqrt{80})^2$$

$$160=80+80$$

Therefore, a right isosceles triangle

Determining the equation of a line



To simplify this section it is best to examine each possible scenario and these include the following:

a) knowing the slope "**m**" and the y-intercept "**b**"

b) knowing the slope "**m**" and the **point** containing the y-intercept "**b**"

c) knowing the slope "**m**" and a **point (x, y)** on the line.

d) knowing two **points** (x_1, y_1) and (x_2, y_2)

e) knowing a **linear equation** and **a point** on a second line and determining the equation of the second line when it is is parallel to or perpendicular to the first line.

f) knowing **two points** on the first line and a **point** on a second line and determining the equation of the second line when it parallel to or perpendicular to the first line

g) knowing the **endpoints** of a line segment and finding the equation of its perpendicular bisector.

H) lines parallel to or perpendicular to the **"x"** or **"y"** axis

Knowing the slope "m" and the y-intercept "b"

Given: $m = 5$ and y-intercept is 6 ($b = 6$)

The slope-intercept formula **$y = mx + b$** is the simpliest to use because all that is required is the substitution of the "m" and "b" by the given values and the result would be the appropriate equation.

$$y = mx + b \Longrightarrow y = 5x + 6$$

Two areas of concern:

a) to write the equation in the form $Ax + By + C = 0$ (standard form)

b) to write the equation with integral coefficients (no fractions)

Given: $m = 3/5$ and the y-intercept is -2 ($b = -2$)

Standard Form and with Integral Coefficients

$$y = mx + b$$
$$y = 3/5\ x - 2$$
$$y - 3/5\ x + 2 = 0$$
$$5*(-3/5)x + 5*\ y + 5 * 2 = 0$$
$$-3x + 5y + 10 = 0$$

To remove a fraction all that you must do is multiply each term by the value(s) found in the denominator and cancel where appropriate.

Knowing the slope "m" and the point containing the y-intercept "b"

Given: $m = -3/4$ and the point containing the y-intercept is $(0, 5)$

It is essential that you can recognize that the point $(0, 5)$ contains the y-intercept and then you can conclude that $b = 5$. If this presents a problem, then I suggest that you consider using the approach covered in the next slide.

Using the slope - intercept form:

$m = -3/4$ and $b = 5$

$$y = mx + b$$

$$y = -3/4x + 5 \Longrightarrow \text{direct substitution}$$

$$3/4\,x + y - 5 = 0 \Longrightarrow \text{standard form}$$

$$4 * 3/4\,x + 4 * y - 4 * 5 = 0$$

$$3x + 4y - 20 = 0 \Longrightarrow \text{integral coefficients}$$

Knowing the slope “m” and a point (x, y) on the line

Given: m = 5 and the line passes through the point (-3, 6)

a) We could substitute the given values into the slope-intercept formula ($y = mx + b$) and find the value of ‘b”

$$y = mx + b$$
$$6 = 5(-3) + b$$
$$6 = -15 + b$$
$$21 = b$$

Using the given information (m= 5) and the calculated b (b = 21) we can re-use the slope-intercept formula and write the equation

$$y = mx + b$$
$$y = 5x + 21$$

b) Using a new formula which allows for direct substitution and simplification

The Point -Slope Formula

Remember the slope formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ if $(x_2 - x_1) \neq 0$

and since $(x_2 - x_1) \neq 0$

we can multiply both sides by the denominator and get the resulting point-slope formula

$$(x_2 - x_1)m = (x_2 - x_1)\frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$(y_2 - y_1) = m(x_2 - x_1) \longleftarrow \boxed{\text{Point-Slope formula}}$$

Re-doing the previous example with m = 5 and the point (-3, 6)

$$(y_2 - y_1) = m(x_2 - x_1)$$

$$(y - 6) = 5(x - (-3))$$

$$y - 6 = 5x + 15$$

$$y = 5x + 21$$

Notes:

a) replace x_2 and y_2 with x and y

b) replace x_1 and y_1 with the x and y values from the given point

c) replace m with the given slope

d) simplify

Assignment:

Review of the formulas $y = mx + b$ and $(y_2 - y_1) = m(x_2 - x_1)$

Determine the equation of the linear function given the following information: (answers in standard form with no fraction coefficients)

a) $m = 4$, $b = 7$

b) $m = -3$, $b = 5$

c) $m = -2/3$, $b = 8$

d) $m = 5/7$, $b = 3/7$

e) $m = 2$, $(0, 8)$

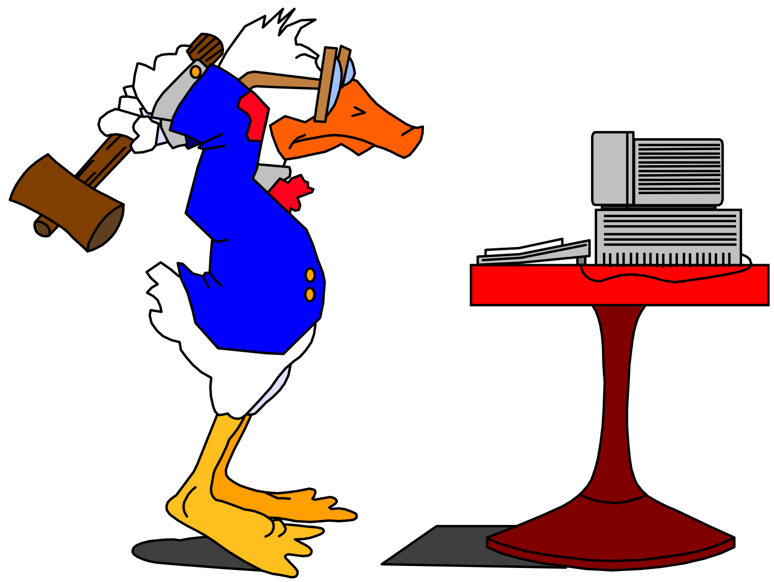
f) $m = -3/4$, $(0, -3)$

g) $m = -2$, $(3, 6)$

h) $m = 5$, $(-2, -6)$

i) $m = -2/5$, $(-3, 7)$

j) $m = 5/4$, $(-2, -3)$



Answer Key:

Questions a - f can be determined using $y = mx + b$

a) $y = 4x + 7$

b) $y = -3x + 5$

c) $y = -2/3\ x + 8$

$3y = -2x + 24$

d) $y = 5/7\ x + 3/7$

$7y = 5x + 3$

e) $y = 2x + 8$

f) $y = -3/4\ x - 3$

$4y = -3x - 12$

Questions g - j can be determined using $(y_2 - y_1) = m(x_2 - x_1)$

g) $(y - 6) = -2(x - 3)$

$y = -2x + 12$

h) $(y - (-6)) = 5(x - (-2))$

$(y + 6) = 5(x + 2)$

$y = 5x + 4$

I) $(y - 7) = -2/5(x - (-3))$

$(y - 7) = -2/5(x + 3)$

$5y - 35 = -2x - 6$

$5y = -2x + 29$

j) $(y - (-3)) = 5/4(x - (-2))$

$(y + 3) = 5/4(x + 2)$

$4y + 12 = 5x + 10$

$4y = 5x -2$

Knowing two points (x_1 , y_1) and (x_2, y_2)

Given: (-3, 6) and (7, -5)

Note: all the previous examples had one common element and that was that we were given the slope. Every question that you are given must provide you with a means to determine (solve an equation for one positive ‘y’-- y = mx + b or to calculate the slope using the slope formula.

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$m = \frac{(6 - (-5))}{(-3 - 7)}$$

$$m = \frac{6 + 5}{-3 - 7}$$

$$m = -\frac{11}{10}$$

$$(y_2 - y_1) = m(x_2 - x_1)$$

$$(y - 6) = \frac{-11}{10}(x - (-3))$$

Remove the denominator by multiplying both sides of the equation by that value

$$10*(y - 6) = 10*\frac{-11}{10}(x - (-3))$$

$$10y - 60 = -11x - 33$$

$$10y = -11x + 27$$

Knowing a linear equation and a point on a second line and determining the equation of the second line when it is is parallel to or perpendicular to the first line.

Example #1:

Given: linear function 5x + 3y = 6 and the point (-2,5) on a second linear function parallel to the first equation.

Step1: Determine the slope

5x + 3y = 6

3y = -5x + 6

y = -5/3 x + 6/3

m_1 = -5/3

m_2 = -5/3

since $m_1 = m_2$ when lines are parallel

Step 2: Substitute determined m and given point into point-slope formula

$$(y_2 - y_1) = m(x_2 - x_1)$$

$$(y-5) = \frac{-5}{3}(x-(-2))$$

$$3*(y-5) = 3*\frac{-5}{3}(x-(-2))$$

$$3y - 15 = -5x - 10$$

$$3y = -5x + 5$$

Example #2:

Given: linear function 5x + 3y = 6 and the point (-2,5) on a second linear function perpendicular to the first equation.

Step1: Determine the slope

$5x + 3y = 6$

$3y = -5x + 6$

$y = -5/3\ x + 6/3$

$m_1 = -5/3$

$m_2 = 3/5$

since $m_1 * m_2 = -1$ when lines are perpendicular

Step 2: Substitute determined m and given point into point-slope formula

$$(y_2 - y_1) = m(x_2 - x_1)$$

$$(y - 5) = \frac{3}{5}(x - (-2))$$

$$5 * (y - 5) = 5 * \frac{3}{5}(x - (-2))$$

$$5y - 25 = 3x + 6$$

$$5y = 3x + 31$$

Knowing two points on the first line and a point on a second line and determining the equation of the second line when it is parallel to or perpendicular to the first line

Example #1:

Given: the points (4, 7) and (9, -1) on the first linear function and the point (-5,-4) on the second linear function parallel to the first.

Step 1: Calculate "m" using the slope formula

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$m = \frac{(7 - (-1))}{(4 - 9)}$$

$$m = \frac{7 + 1}{4 - 9}$$

$$m = -\frac{8}{5}$$

m_1=-8/5 → m_2 = -8/5

Step 2: Substitute determined m and given point into point-slope formula

$$(y_2 - y_1) = m(x_2 - x_1)$$

$$(y - 4) = \frac{-8}{5}(x - (-5))$$

$$5*(y - 4) = 5*\frac{-8}{5}(x - (-5))$$

$$5y - 20 = -8x - 40$$

$$5y = -8x - 20$$

Example #2:

Given: the points (4, 7) and (9, -1) on the first linear function and the point (-5,-4) on the second linear function perpendicular to the first.

Step 1: Calculate "m" using the slope formula

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$m = \frac{(7 - (-1))}{(4 - 9)}$$

$$m = \frac{7 + 1}{4 - 9}$$

$$m = -\frac{8}{5}$$

Since lines perpendicular $m_1 * m_2 = -1$, therefore if $m_1 = -8/5$ then $m_2 = 5/8$

Step 2: Substitute determined m and given point into point-slope formula

$$(y_2 - y_1) = m(x_2 - x_1)$$

$$(y - 4) = \frac{5}{8}(x - (-5))$$

$$8 * (y - 4) = 8 * \frac{5}{8}(x - (-5))$$

$$8y - 32 = 5x + 25$$

$$8y = 5x + 57$$

Knowing the endpoints of a line segment and finding the equation of its perpendicular bisector.

Given: the endpoints of a line segment are (-5, -3) and (7, 5)

Step 1: Determine "m"

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$m = \frac{(-3 - 5)}{(-5 - 7)}$$

$$m = \frac{-3 - 5}{-5 - 7}$$

$$m = \frac{8}{12} = \frac{2}{3}$$

Since lines perpendicular $m_1 * m_2 = -1$, therefore if $m_1 = 2/3$ then $m_2 = -3/2$

Step 2: Determine the midpoint

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M\left(\frac{-5+7}{2}, \frac{-3+5}{2}\right)$$

$$M\left(\frac{2}{2}, \frac{2}{2}\right)$$

$$M(1,1)$$

Step 3: Combine the information from steps 1 and 2 (m = -3/2 and the point is (1, 1)) and substitute those values into the point slope formula

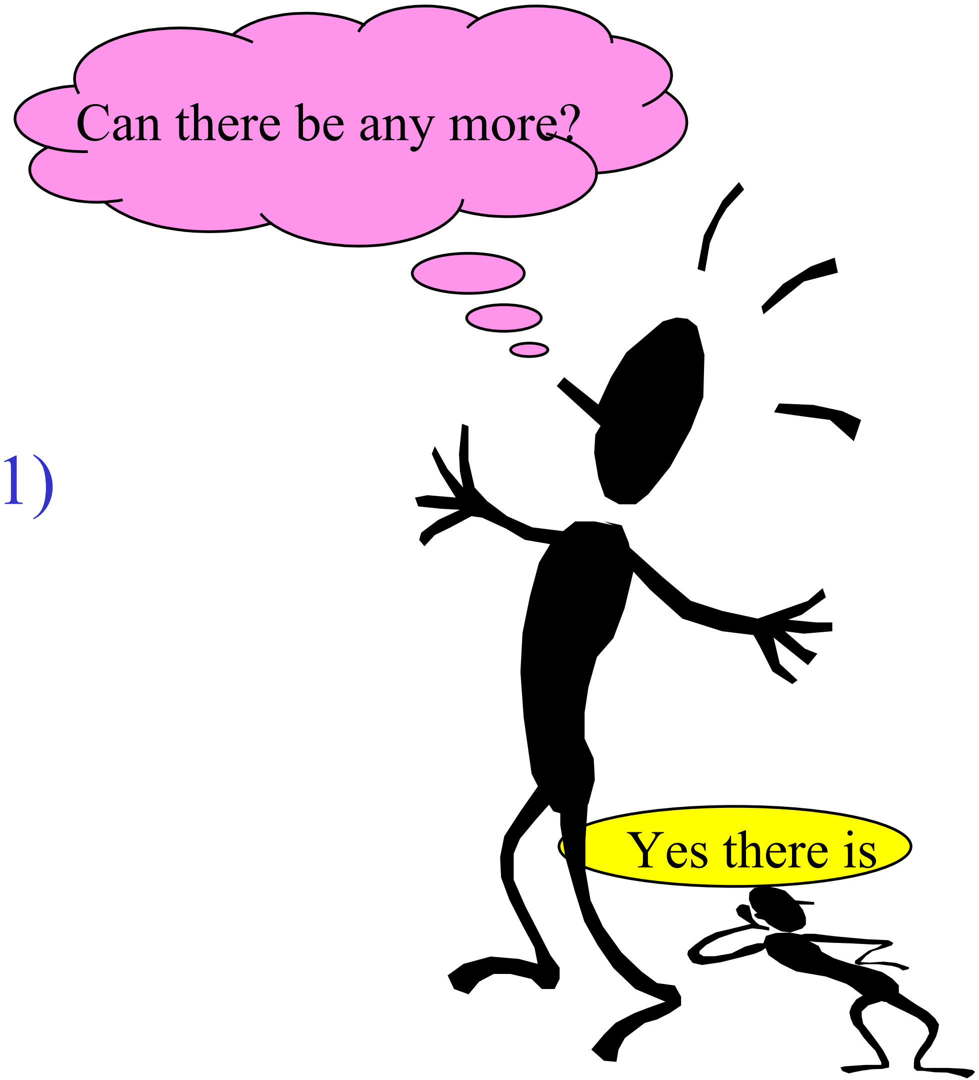
$$\left(y_2 - y_1\right) = m\left(x_2 - x_1\right)$$

$$(y - 1) = \frac{-3}{2}(x - 1)$$

$$2 * (y - 1) = 2 * \frac{-3}{2}(x - 1)$$

$$2y - 2 = -3x + 3$$

$$2y = -3x + 5$$



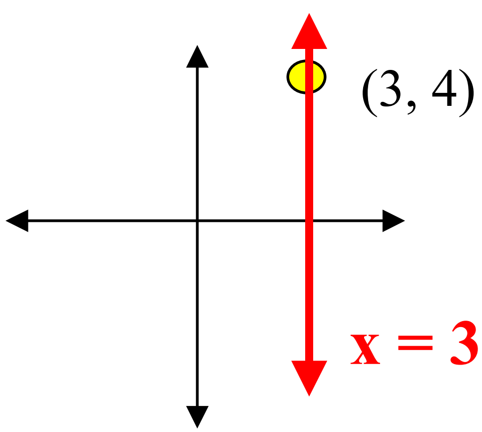
Knowing the the lines orientation to the axis on a coordinate plane

Orientation to the y-axis:

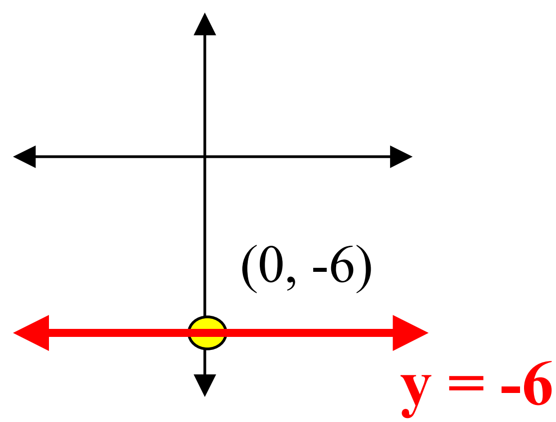
a) If a line is parallel to the y-axis it would be a vertical line and be represented by the equation x = k (k is a constant).

b) If a line is perpendicular to the y-axis it would be a horizontal line and be represented by the equation y = k (k is a constant)

Example #1:
passing through (3, 4) and parallel to the y-axis



Example #2:
having a y-intercept of -6 and perpendicular to the y-axis



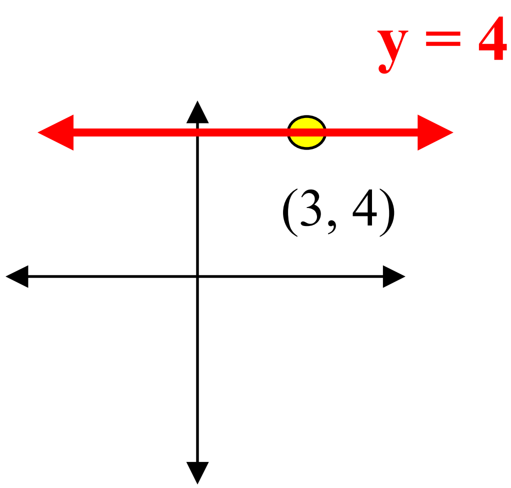
Orientation to the y-axis:

a) If a line is parallel to the x-axis it would be a horizontal line and be represented by the equation y = k (k is a constant).

b) If a line is perpendicular to the x-axis it would be a vertical line and be represented by the equation x = k (k is a constant)

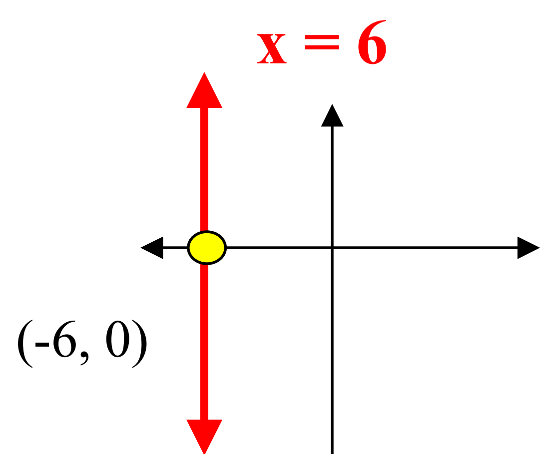
Example #1:

passing through (3, 4) and parallel to the x-axis

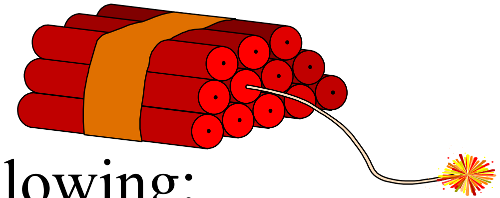


Example #2:

having a y-intercept of -6 and perpendicular to the y-axis



Assignment



Determine the equation of the line given the following:

a) through the points (5, 3) and (-4, 1)

b) through the points (-7, -3) and (-2, -9)

c) through the point (5, 1) and parallel to the graph $3x + 7y = 6$

d) through the point (-3, -2) and perpendicular to the graph of $-5x + 2y = 7$

e) through the point (6, -1) and parallel to the line passing through the points (-3, 7) and (9, -3)

f) through the point (-1, 7) and perpendicular to the line passing through the points (-9, 0) and (0, -7)

g) is a perpendicular bisector of the line segment defined by the points (-3, 7) and (9, -5)

h) passing through (-5, 2) and parallel to the x - axis

I) passing through the x-intercept 7 and perpendicular to the x-axis.

Answer key:

a.

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$m = \frac{(3 - 1)}{(5 - (-4))}$$

$$m = \frac{2}{9}$$

$$(y_2 - y_1) = m(x_2 - x_1)$$

$$(y - 3) = \frac{2}{9}(x - 5)$$

$$9y = 2x + 17$$

b.

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$m = \frac{((-3) - (-9))}{((-7) - (-2))}$$

$$m = \frac{6}{-5} = -\frac{6}{5}$$

$$(y_2 - y_1) = m(x_2 - x_1)$$

$$(y - (-3)) = -\frac{6}{5}(x - (-7))$$

$$5y = -6x - 57$$

c. 3x + 7y = 6

7y = -3x + 6

y = -3/7 x + 6/7

m_1 = -3/7

m_2 = -3/7

$$(y_2 - y_1) = m(x_2 - x_1)$$

$$(y - 1) = -\frac{3}{7}(x - 5)$$

$$7y = -3x + 22$$

d. 5x + 2y = 7

2y = 5x + 7

y = 5/2 x + 7/2

m_1 = 5/2

m_2 = -2/5

$$(y_2 - y_1) = m(x_2 - x_1)$$

$$(y - (-2)) = -\frac{2}{5}(x - (-3))$$

$$5y = -2x - 16$$

e. $$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$m = \frac{(7 - (-3))}{((-3) - 9)}$$

$$m = \frac{10}{-12} = -\frac{5}{6}$$

$$(y_2 - y_1) = m(x_2 - x_1)$$

$$(y - (-1)) = -\frac{5}{6}(x - 6)$$

$$6y = -5x + 24$$

f.

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$m = \frac{(0 - (-7))}{((-9) - 0)}$$

$$m = \frac{7}{-9} = -\frac{7}{9}$$

$$m_2 = \frac{9}{7}$$

$$(y_2 - y_1) = m(x_2 - x_1)$$

$$(y - 7) = \frac{9}{7}(x - (-1))$$

$$7y = 9x + 58$$

g.

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$m = \frac{(7 - (-5))}{((-3) - 9)}$$

$$m = \frac{12}{-12} = -1$$

$$m_2 = 1$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M\left(\frac{-3+9}{2}, \frac{7+(-5)}{2}\right)$$

$$M\left(\frac{6}{2}, \frac{2}{2}\right)$$

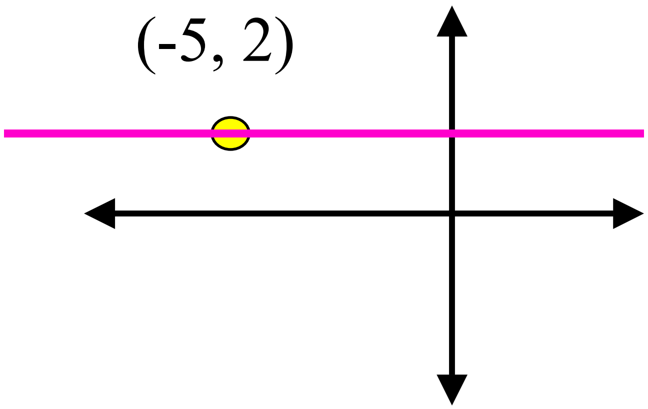
$$M(3,1)$$

$$(y_2 - y_1) = m(x_2 - x_1)$$

$$(y - 1) = 1(x - 3)$$

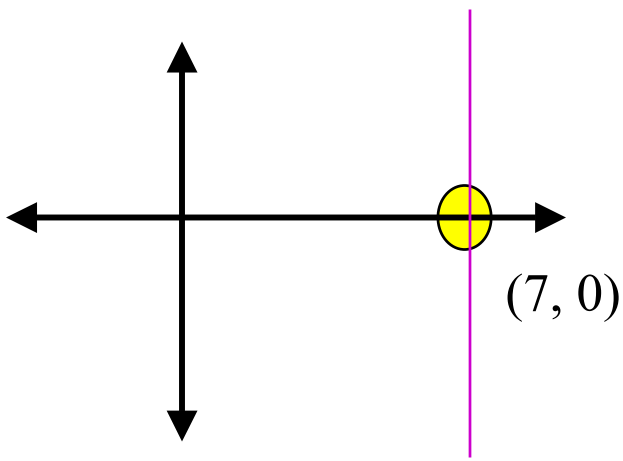
$$y = x - 2$$

h.



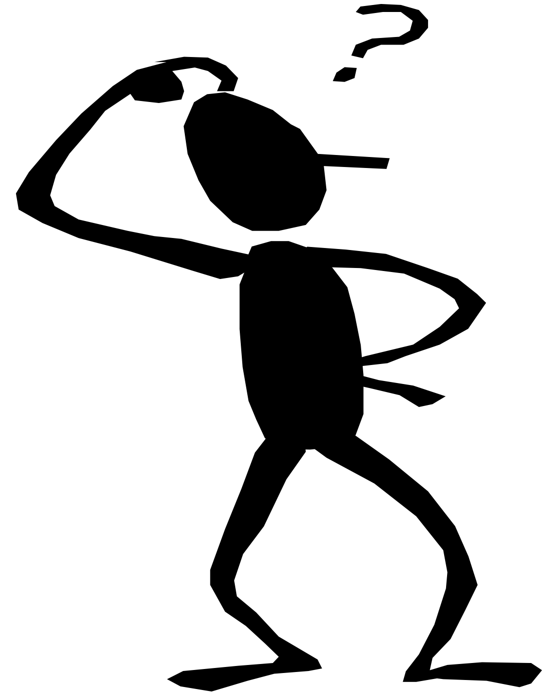
Zero slope: equation y = 2

i.



No slope: equation x = 7

Review



1. Identify the formula for:
   a) slope knowing two points
   b) midpoint formula
   c) distance formula
   d) slope-intercept formula
   e) point-slope formula

2. For the set of points (-3, 7) and (-5, 1) determine:
   a) the slope of the line defined by the two points
   b) the midpoint of the line segment defined by the two points
   c) the distance between the two points
   d) the slope of a line parallel to the line defined by the two points
   e) the slope of a line perpendicular to the line defined by the two points

3. Given the linear equation $4x + 3y = 6$, determine

a) the slope of the line

b) the value of the y-intercept

c) the value of the x-intercept

4. Determine the equation of the line given the following information as it pertains to the given line

a) $m = 4$, $b = -7$

b) $m = 3/5$, $(0, 6)$

c) $m = 7/2$, $(-2, 5)$

d) $(5, -2)$ $(4, 9)$

e) passing through $(-3, -4)$ and parallel to the line with equation $5x - 4y = 2$

f) passing through $(2, 5)$ and perpendicular to the line passing through the points $(-3, 8)$ and $(4, -2)$

g) perpendicular bisector of line segment defined by the points $(9, 4)$ and $(-7, -2)$

h) parallel to the y-axis passing through $(-7, 5)$

5. Given the points (4, 5) (-3, 7) and (9, 3)

a) Are the given points collinear? If you answer is no, go to part "b"

b) Since the given points represent the vertices of a triangle determine whether the triangle is isosceles, equilateral, or scalene.

c) If scalene or isosceles determine whether the triangle is also a right triangle.

d) Determine the perimeter of the triangle

6. What type of lines are represented by these equations?

$5x - 3y = 8$ and $-10x - 6y = 12$

Answer key

1. a) $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

b) $M\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

c) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

d) $y = mx + b$

e) $(y_2 - y_1) = m(x_2 - x_1)$

2. a) $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{7-1}{(-3)-(-5)} = \frac{6}{2} = 3$$

b) $M\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

$$M\left(\frac{-3+(-5)}{2}, \frac{7+1}{2}\right)$$

$$M\left(-\frac{8}{2}, \frac{8}{2}\right) = (-4,4)$$

c) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$d = \sqrt{((-3)-(-5))^2 + (7-1)^2}$$

$$d = \sqrt{2^2 + 6^2} = \sqrt{40} = 2\sqrt{10}$$

d) m = 3

e) m = -1/3

3. y = -4/3 x + 6/3

a) m = -4/3

b) b = 2

c) 4x + 3(0) = 6

x = 6/4 = 3/2

4. a) y = mx + b

y = 4x + 7

b) y = mx + b

y = 3/5 x + 6

5y = 3x + 30

c) $(y_2 - y_1) = m(x_2 - x_1)$

$$(y-5) = \frac{7}{2}(x-(-2))$$

$$2y = 7x + 24$$

d) $$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-9}{5-4} = \frac{-11}{1} = -11$$

$$(y_2 - y_1) = m(x_2 - x_1)$$

$$(y-(-2)) = -11(x-5)$$

$$y = -11x + 53$$

e) $-4y = -5x + 2$

$$y = \frac{5}{4}x - \frac{2}{4}, \qquad m_1 = 5/4, m_2 = 5/4$$

$$(y_2 - y_1) = m(x_2 - x_1)$$

$$(y-(-4)) = -\frac{5}{4}(x-(-3))$$

$$4y = -5x - 23$$

f) $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$\frac{8-(-2)}{-3-4} = \frac{10}{-7} = -\frac{10}{7}$$

$$m_2 = 7/10$$

$$(y_2 - y_1) = m(x_2 - x_1)$$

$$(y-5) = \frac{7}{10}(x-2)$$

$$10y = 7x + 36$$

g) $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$\frac{4-(-2)}{9-(-7)} = \frac{6}{16} = \frac{3}{8}$$

$$m_2 = -8/3$$

g) cont'd $M\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

$$M\left(\frac{9+(-7)}{2}, \frac{4+(-2)}{2}\right)$$

$$M\left(\frac{2}{2}, \frac{2}{2}\right) = (1,1)$$

$$(y_2 - y_1) = m(x_2 - x_1)$$

$$(y-1) = -\frac{8}{3}(x-1)$$

$$3y = -8x + 9$$

h) no slope: equation x = -7

5.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{5-7}{4-(-3)}$$
$$m = \frac{-2}{7} = -\frac{2}{7}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{5-3}{4-9}$$
$$m = \frac{2}{-5} = -\frac{2}{5}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{7-3}{-3-9}$$
$$m = \frac{4}{-12} = -\frac{1}{3}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(4-(-3))^2 + (5-7)^2}$$
$$d = \sqrt{7^2 + (-2)^2} = \sqrt{53}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(4-9)^2 + (5-3)^2}$$
$$d = \sqrt{(-5)^2 + 2^2} = \sqrt{29}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{((-3)-9)^2 + (7-3)^2}$$
$$d = \sqrt{(-12)^2 + 4^2} = \sqrt{160}$$

a) since slopes different points are not collinear
b) since all sides different the triangle is scalene
c) since no slopes are negative reciprocals of one another the triangle is not a right triangle
d) perimeter = $\sqrt{160} + \sqrt{53} + \sqrt{29}$

6. $-3y = -5x + 8$

$y = -5/-3\ x + 8/-3$

$m = 5/3,\ b = -8/3$

$-6y = -10x + 12$

$y = -10/-6 + 12/-6$

$m = 10/6 = 5/3,\ b = 12/-6 = -2$

since the slopes are the same the the y-intercepts different the two lines are parallel