Linear Functions One

Contents

Students will

- learn the definition of a coordinate plane, slope, x and y - intercepts
- graph a linear function using one of three methods:
  - table of values
  - slope and y-intercept
  - x and y intercept
- recognize special categories of lines and be able to draw these lines

The Coordinate Plane and Points

In the coordinate plane, or the rectangular coordinate system, the vertical y-axis and the horizontal x-axis intersect at a point called the origin.

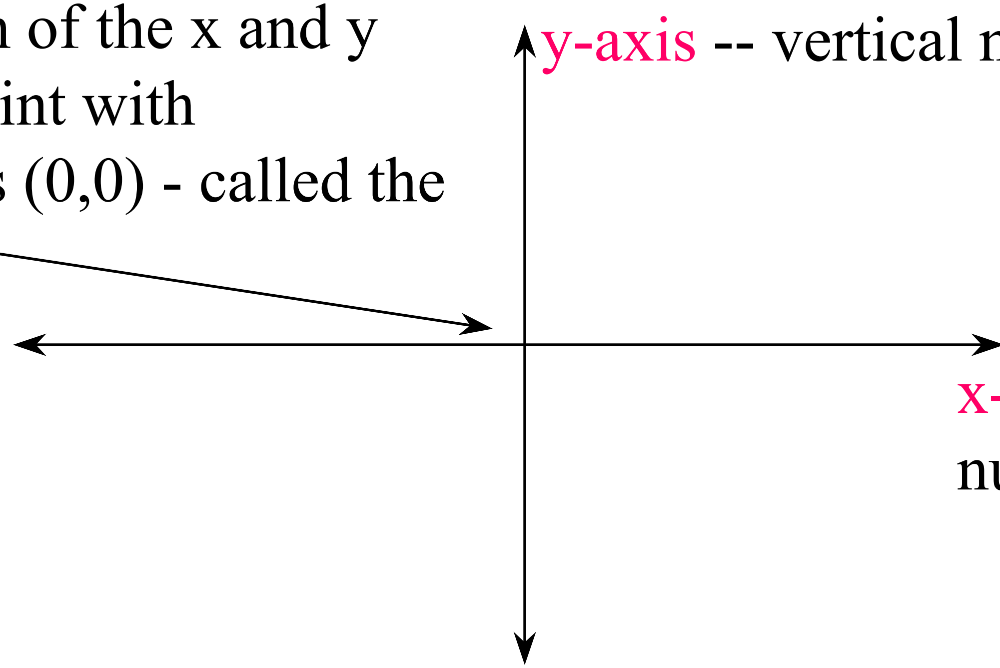
This section will help you better understand the coordinate plane and how to graph points on the plane.

Coordinate Plane

2 dimensional plane -- a sheet of graph paper

intersection of the x and y axis is a point with coordinates (0,0) - called the origin

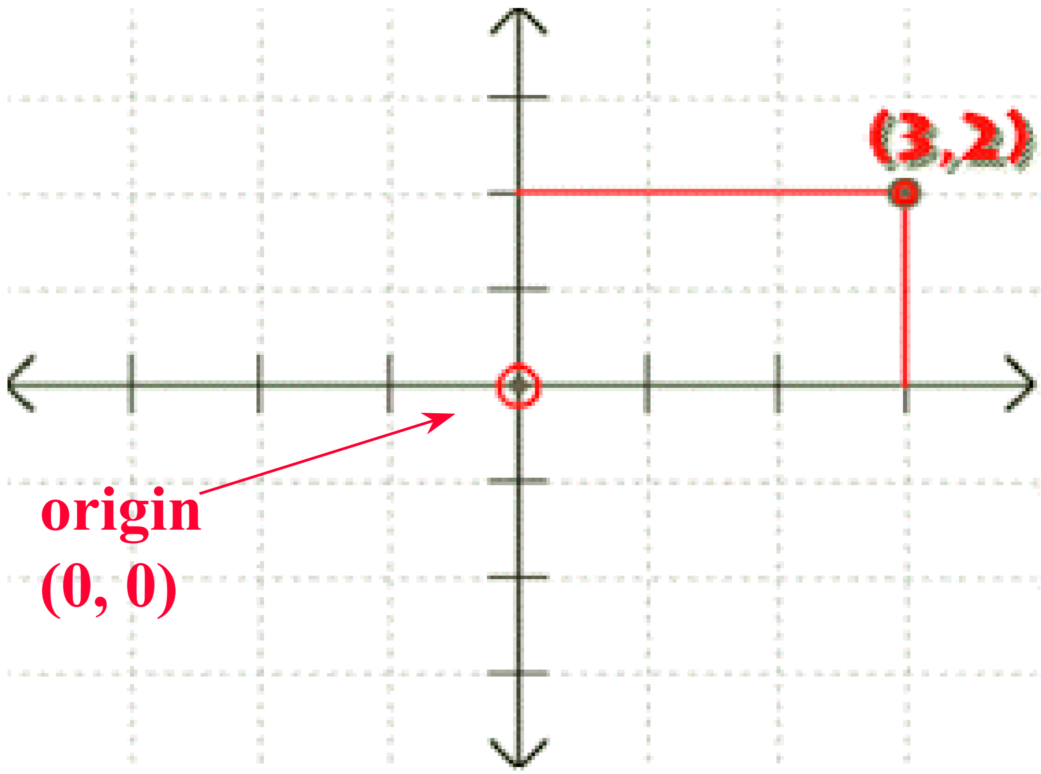
y-axis -- vertical number line



x-axis -- horizontal number line

Important Things To Remember

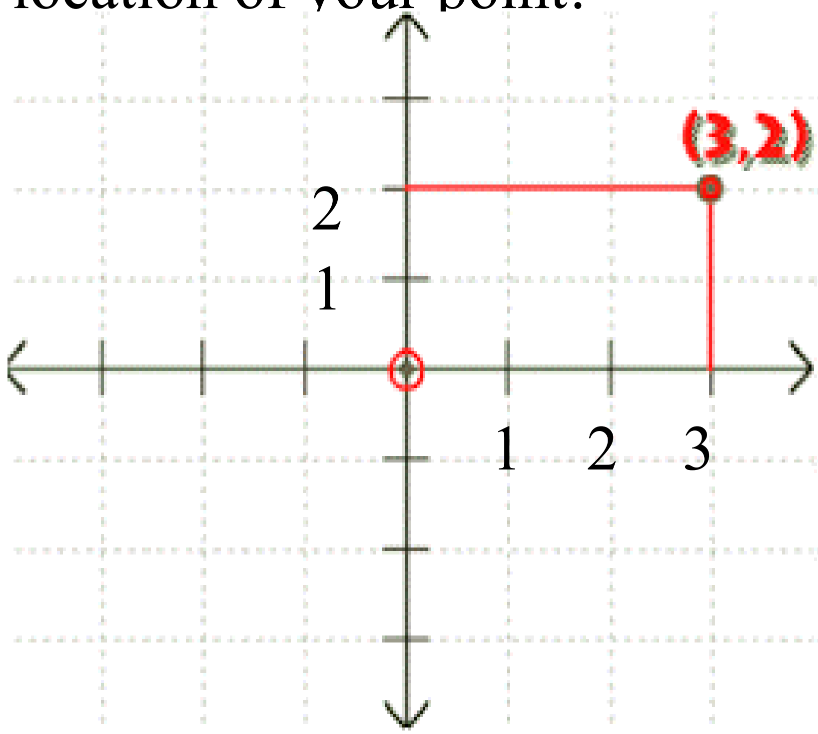
The origin's coordinates are (0,0).
Points are named by an ordered pair. Example: (3,2)
The first number in an ordered pair is the x-coordinate, and the second number listed is the y-coordinate. Example:



The Tutorial

When graphing, the coordinate plane will be labeled with "tick marks" denoting the scale. Beginning at the origin, count along the x-axis scale until you find the tick mark labeled with the x-axis coordinate of your point, and then count along the y-axis scale until you find the tick mark labeled with the y-axis coordinate of your point. That is the location of your point!

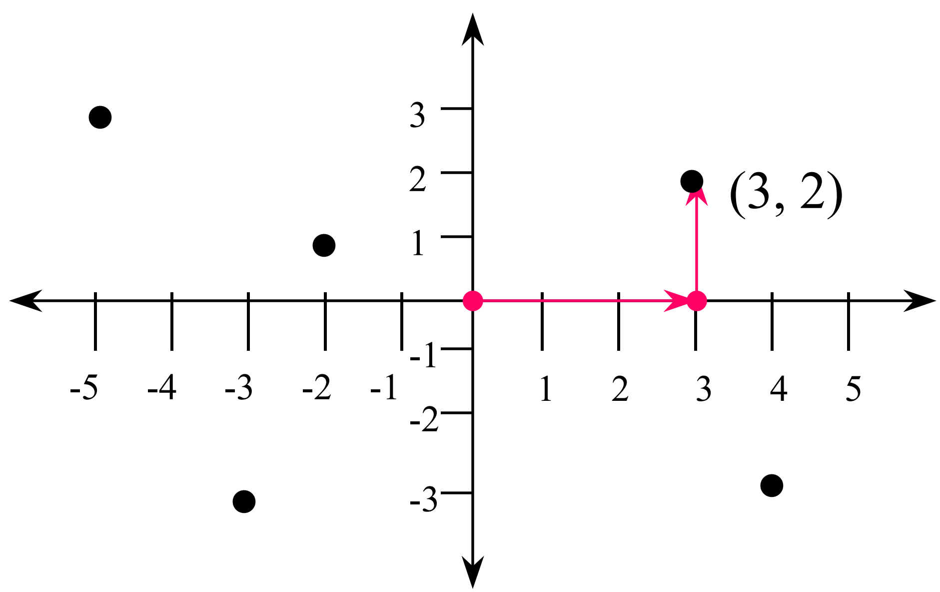
Example



Ordered Pair

Locate the point (3, 2) on the graph below. Start at the origin and count 3 units to the right along the x-axis. From this location count 2 units up and mark the spot and in addition, label the coordinates.

Locate the following points on the graph: (-2, 1), (-3, -3), (4, -3), (-5, 3)



Graphing Linear Equations

Methods that can be used:

a) Table of Values
b) Slope-Intercept Form
c) X and Y Intercept Form

Points of Note:

a) all axis must be drawn using a straight edge
b) all plotted points must be labeled
c) all plotted points must be joined using a straight edge
d) all graphs must be labeled with the equation name

Graphing Using Ordered Pairs of Numbers - Table of Values

When graphing linear equations, "plugging in points" is a suggested method of solving the equations and putting them in a graphical format. To plug in points, select an x-coordinate (be reasonable in the number you select for the x-coordinate) and put the x-axis coordinate in the equation in place of x. Then solve the equation. This will give you a y-value. Put your chosen x-value and the y-value you solved for together, and you will have an ordered pair (a point) that you can graph. Repeat this process about 4 or 5 times and then connect the points you have graphed. The line you see will be the graph of a linear equation.

Example

1. Graph: $y = 2x + 1$

   Solution:

   $y = 2(0) + 1$

   $y = 1$ — An x-coordinate of 0 was selected.

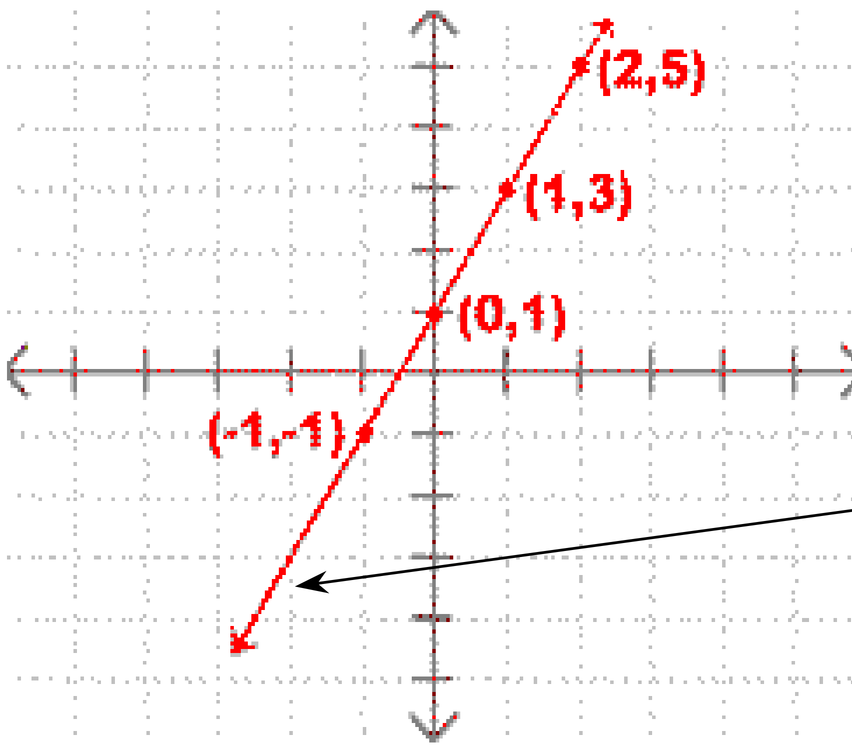
   $(0, 1)$ — The equation was solved for y.

   The resulting ordered pair is (0,1).

x	y
0	1
1	3
2	5
-1	-1

The process described above repeated 4 times. The results are shown to the left in table form

The graph appears as:



The equation of the graph is $y = 2x + 1$

In math, **a line is defined to be of infinite length and consisting of at least 2 points**. All lines are straight (a line is straight - a curve is curved). When you need to graph an equation such as $y = -(1/2)x + 2$, the only thing you need to be especially wary of is the fraction. Since a line consists of two or more points, all you need to do is find two or more ordered pairs that solve the equation. The easiest way to do this is to draw a table such as the following and fill it in:

x	y
0	
2	
4	

You plug the x-values into the equation and find the y-values. That gives you ordered pairs that you can graph on the coordinate plane and then "connect" into a line.

Example

2. Graph: $y = -.5x + 2$

Solution: Begin by making a table (choose convenient values for x).

x	0	2	-2
y			

Now plug the x-values into the original equation and find the values for y.

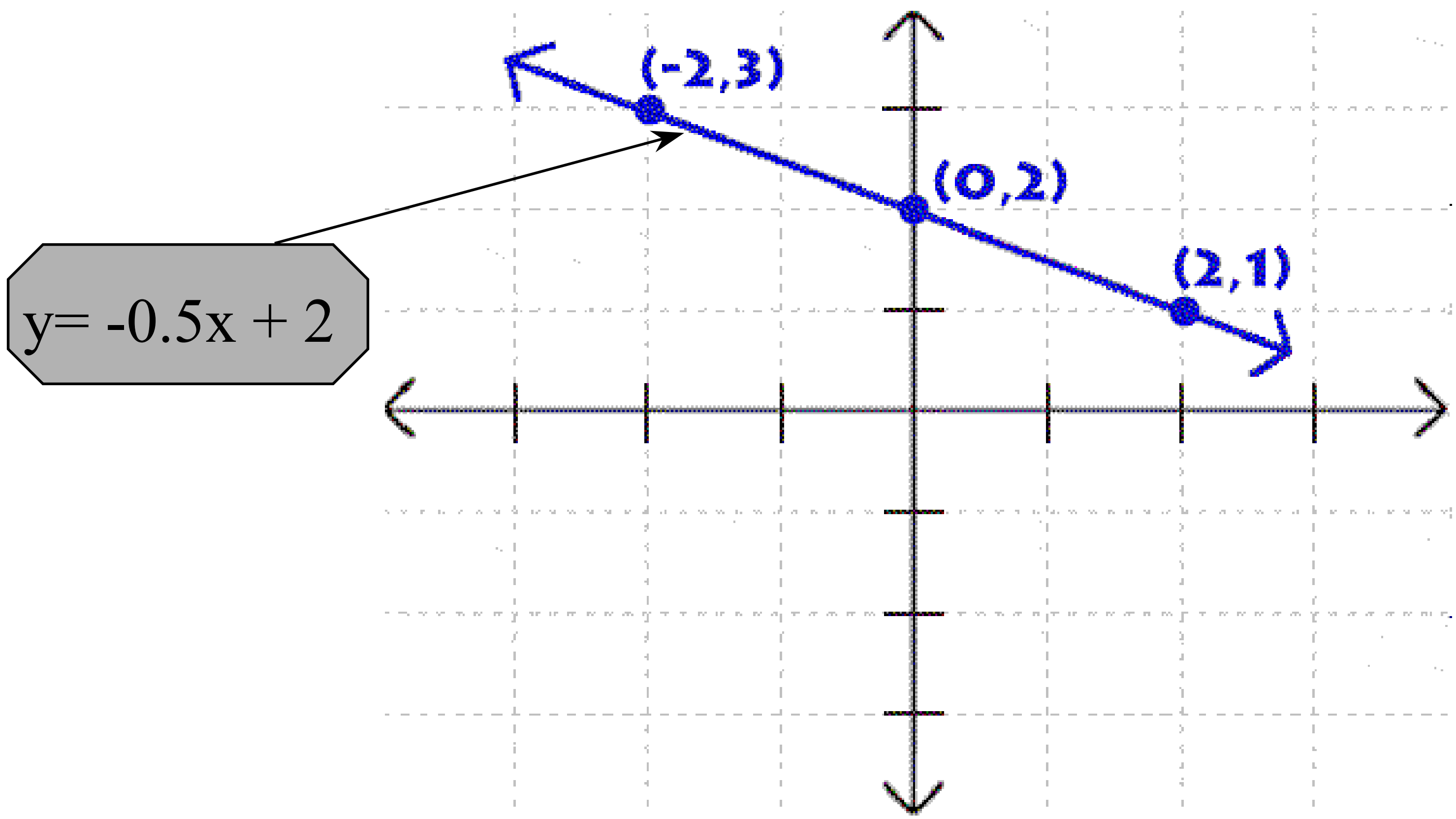
$y = -.5(0) + 2$ $y = 2$ Complete the table.

$y = -.5(2) + 2$ $y = 1$

$y = -.5(-2) + 2$ $y = 3$

x	0	2	-2
y	2	1	3

Now graph the points and draw a line by "connecting the dots." (Aren't you overwhelmed by all this fun?) Here's what it looks like:



Your Turn

Procedure:

a) Create a table of values for each of the following. Use a minimum three values for “x”. Check your answers with the next slide.

b) Graph each set of points, label your graph and then check your answers with the graphs on the slide labeled “Graphic Answers”

a) $2x + y = 6$

b) $4x - 2y = 5$

c) $-3x + y = -2$

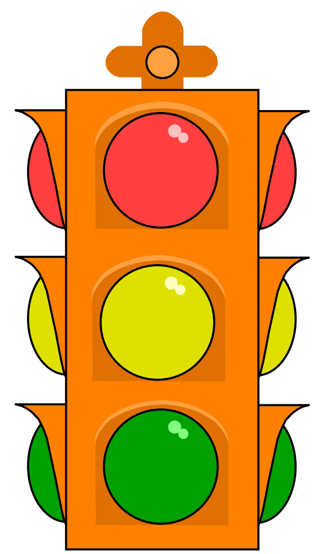


Table of Values for each Question

a)

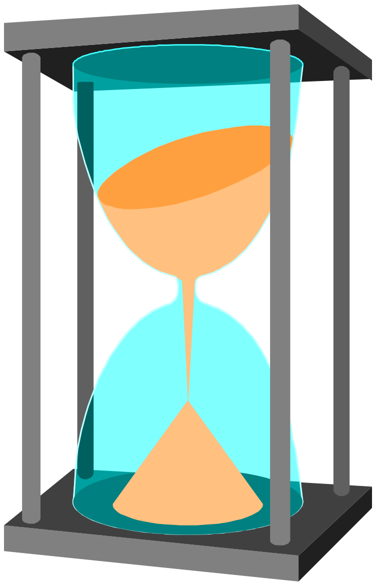
x	-4	-3	-2	-1	0	1	2	3	4
y	14	12	10	8	6	4	2	0	-2

b)

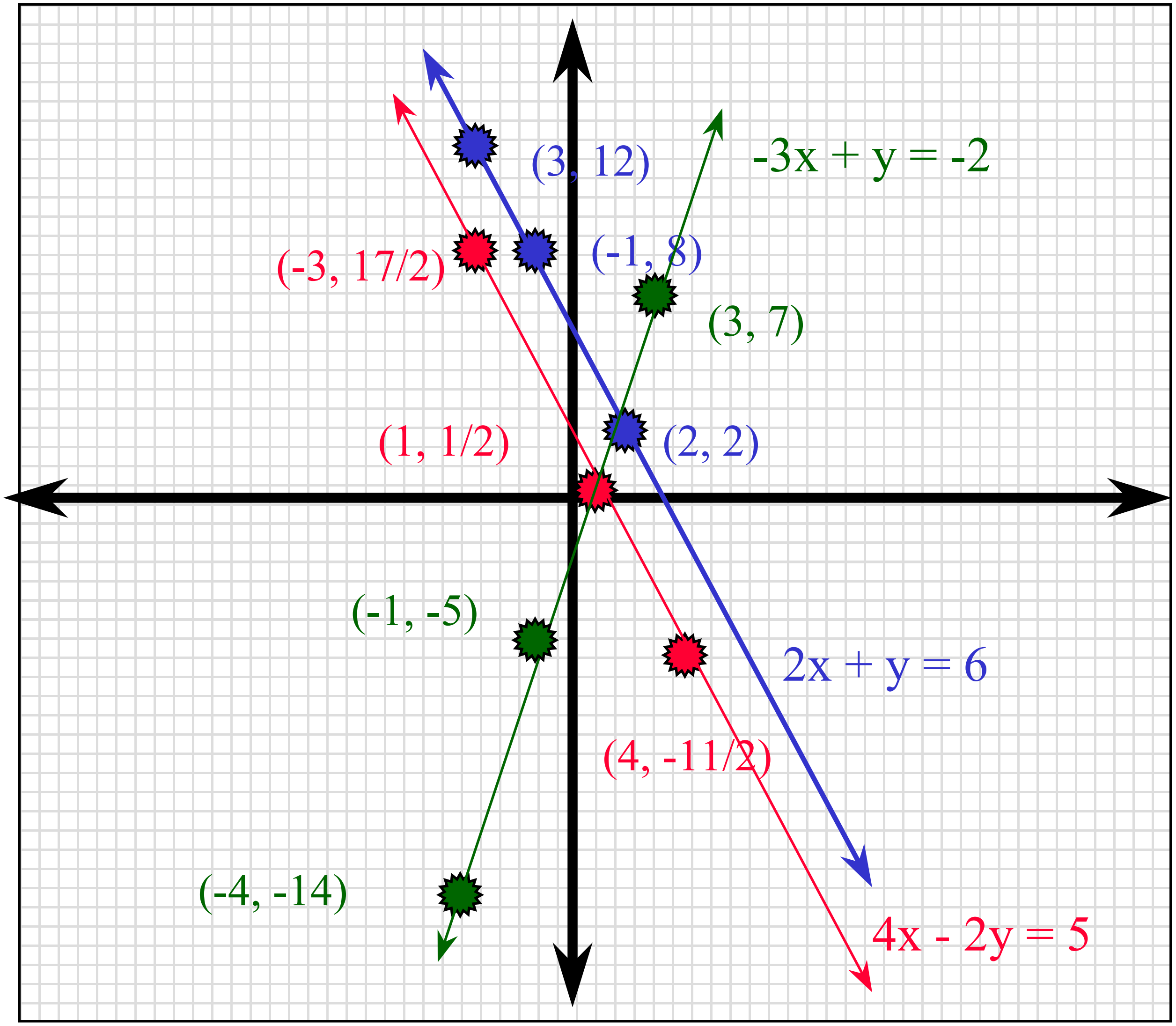
x	-4	-3	-2	-1	0	1	2	3	4
y	$\frac{21}{2}$	$\frac{17}{2}$	$\frac{13}{2}$	$\frac{9}{2}$	$\frac{5}{2}$	$\frac{1}{2}$	$\frac{-3}{2}$	$\frac{-7}{2}$	$\frac{-11}{2}$

c)

x	-4	-3	-2	-1	0	1	2	3	4
y	-14	-11	-8	-5	-2	1	4	7	10



Graphic Answers



Using the Slope Intercept Procedure

In this section you will graph a linear equation using a procedure referred to as slope intercept.

Linear equations can be graphed when in the form $y = mx + b$. This means that you must take any linear equation and solve for one positive "y".

The coefficient of x "m" is now identified as the slope of the line and the constant "b" is called the y-intercept.

Definition of Slope

- slope is defined as a change in vertical distance (rise) divided by change in horizontal distance (run)
- examples: ski run (blue, green, black diamond), house roof, ramp,
- the symbol for slope is m and the formula is

$$m = \frac{rise}{run}$$

Plotting Points Using Slope

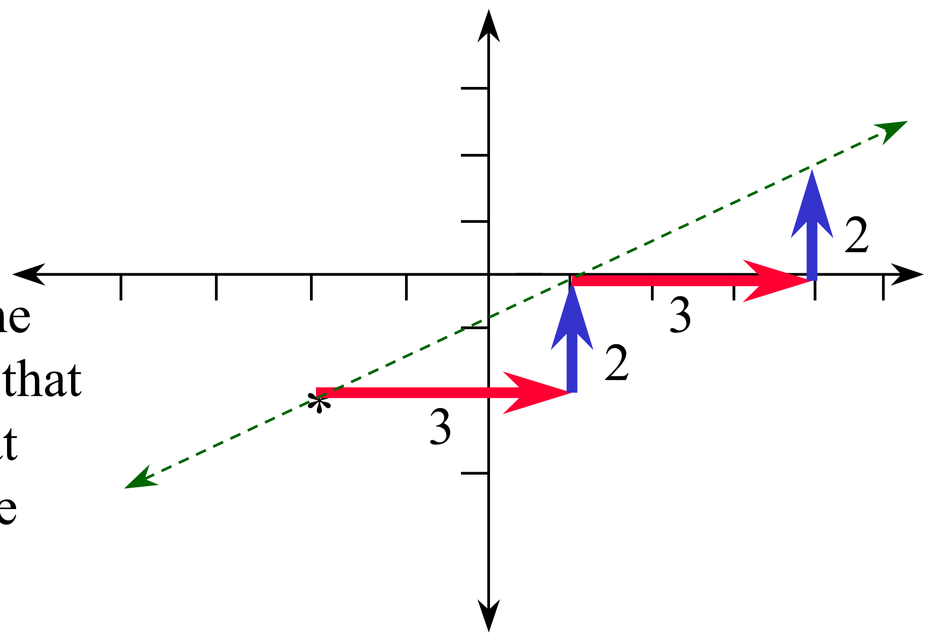
Note: run will always be considered as horizontal movement from a point and it will always be in a right direction (forward from the point). Therefore, run will always be considered as positive

rise will only be considered after the run has been counted.
From this location you will count up if the slop is positive and down if the slope is negative

Example 1:

m = 2/3 starting at (-2, -2)

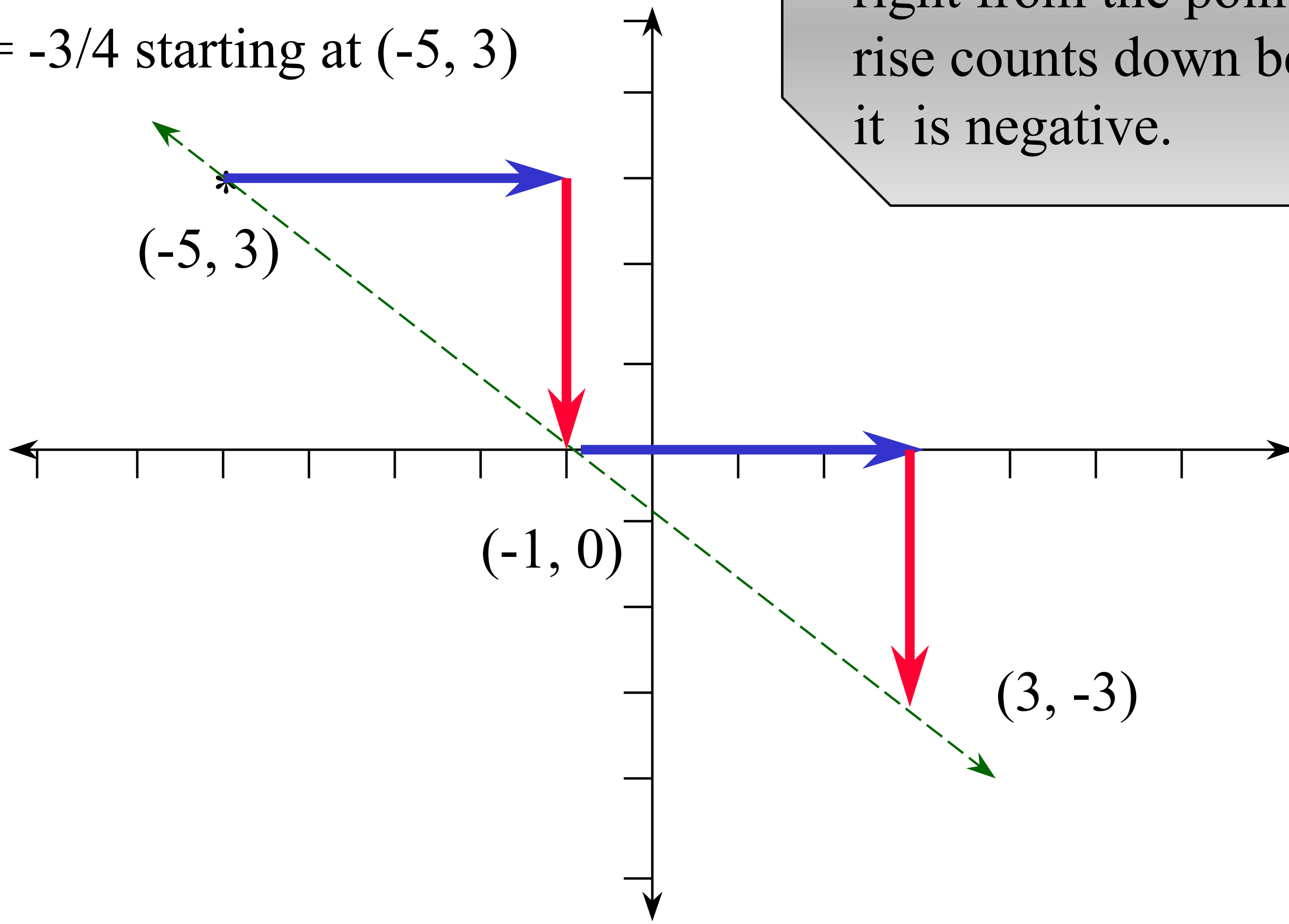
procedure: count over 3 units from the point and then count up 2 units from that location, mark this point (1, 0), repeat procedure from this point locating the point (4, 2)
join the three points



Example 2 :

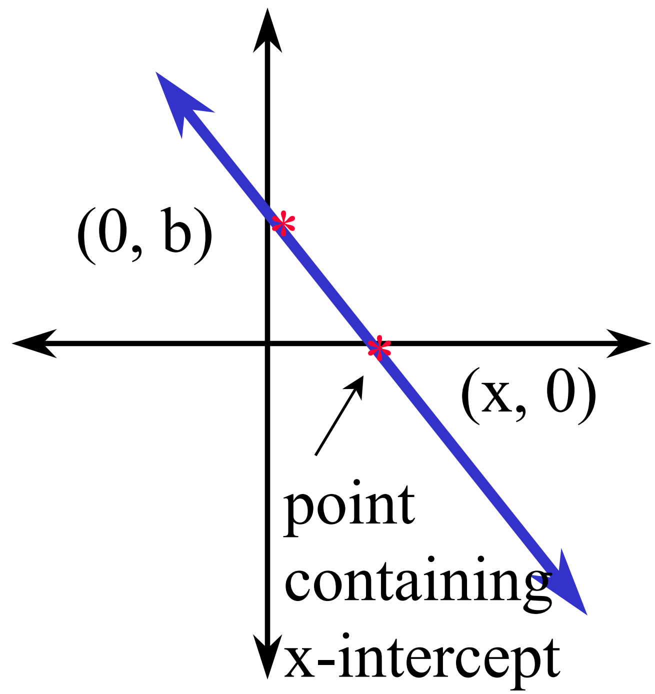
m = -3/4 starting at (-5, 3)

Remember - run counts right from the point and rise counts down because it is negative.



Definition of Intercepts

Points where a line cuts the x and y axis. When a line cuts the y -axis, the x-coordinate of that point has a value of “0” and the y-coordinate is referred to as the y-intercept (some real number usually defined by the letter “b”). The point is designated as (0, b). When a line cuts the x-axis, the x-coordinate of that point is referred to as the x-intercept and has some real value while the y-coordinate has a value of “0”. This is usually designated as (x, 0),



Procedure to Determine the Slope and y-Intercept

- Example $3x + 2y = 6$

Step 1: Isolate the term containing the y-variable — $2y = -3x + 6$

Step 2: Make sure the coefficient of "y" is positive "1" — $y = \frac{-3}{2}x + \frac{6}{2}$

write the right side as two separate fractions which then will parallel the general form $y = mx + b$ and makes identification of the slope and y-intercept easier

Step 3: Identify "m" and "b" and the point containing "b" -- (0, b) — $m = -3/2$, $b = 6/2$ or 3, point (0, 3)

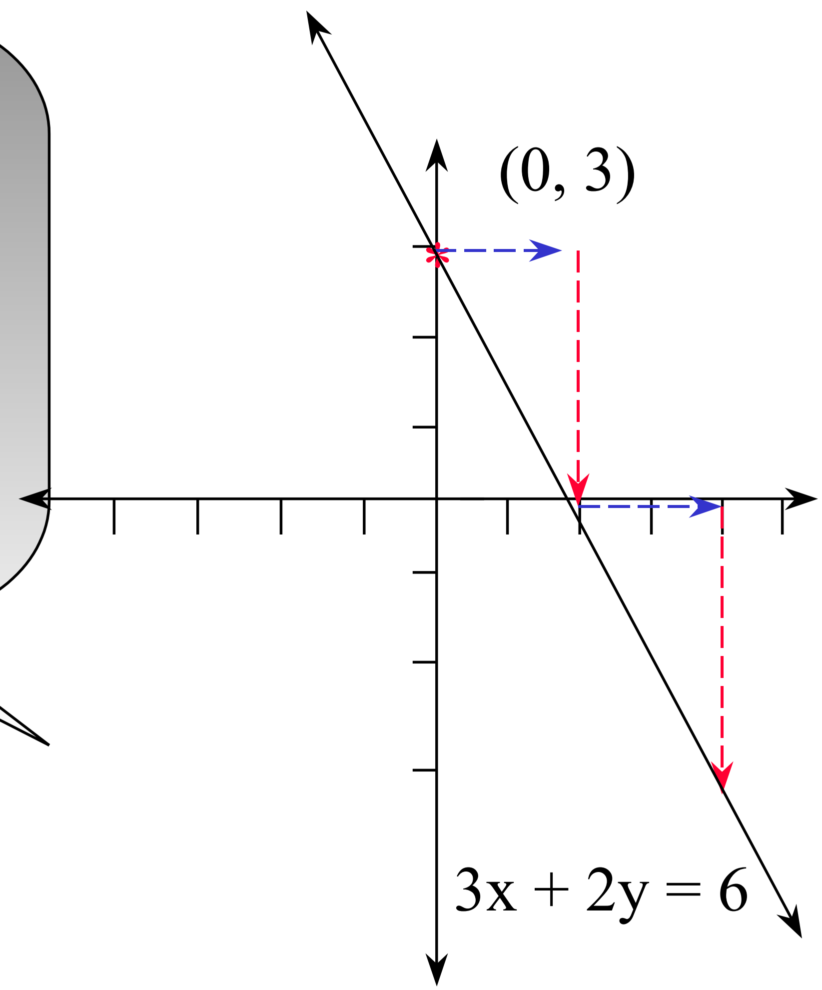
Using Slope and y-Intercept to Graph

Step 1: Plot the point containing the y-intercept. Label this point.

Step 2: Use slope and its definition of rise (red) and run (blue) to determine 2 other points

Step 3: Join the points

Step 4: Label the graph



Your Turn

Procedure:

a) Take each of the following equation and solve for one positive y. Check your answers with slide titled “One Positive Y”

b) Identify the slope (m), y-intercept (b) and point containing the y-intercept for each equation (0, b). Check your answers with slide titled “Necessary Info”.

c) Graph and label each graph. Check your graphs with those on the slide titled “Graphic Answers”

a) $3x + 2y = 6$

b) $-4x + 3y = -12$

c) $5x - 2y = -10$

One Positive Y

a) $3x + 2y = 6$

$$2y = -3x + 6$$

$$y = \frac{-3x}{2} + \frac{6}{2}$$

b) $-4x + 3y = -12$

$$3y = 4x - 12$$

$$y = \frac{4x}{3} - \frac{12}{3}$$

c) $5x - 2y = -10$

$$-2y = -5x - 10$$

$$y = \frac{-5x}{-2} - \frac{10}{-2}$$

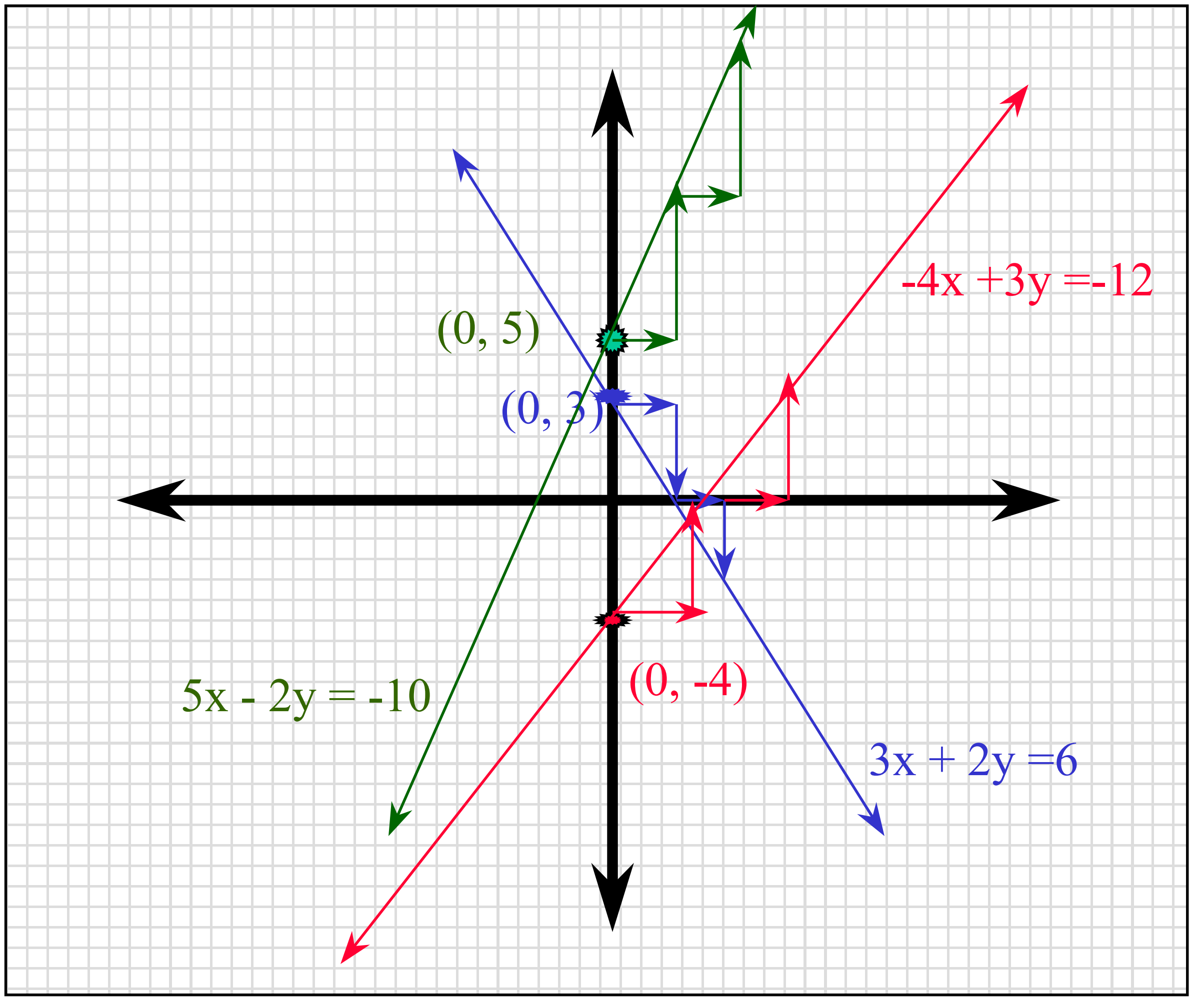
Necessary Info

a) $m = -3/2$
$b = 3$
point $(0, 3)$

b) $m = 4/3$
$b = -4$
point $(0, -4)$

c) $m = 5/2$
$b = 5$
point $(0, 5)$

Graphic Answers



Graphing Using the Intercept Method

This procedure requires the calculation of the points (0, b) which contains the y-intercept and the point (x, 0) which contains the x-intercept.

Example: $5x - 2y = 10$

a) the y-intercept:

- substitute the value "0" for "x"
- solve for one positive "y"
- write solution as a point

$$5(0) - 2y = 10$$
$$0 - 2y = 10$$
$$-2y/-2 = 10/-2$$
$$y = -5$$

Point (0, -5)

b) x-intercept:

- substitute the value "0" for "y"
- solve for one positive "x"
- write solution as a point

$$5x - 2(0) = 10$$
$$5x - 0 = 10$$
$$5x/5 = 10/5$$
$$x = 2$$

point (2, 0)

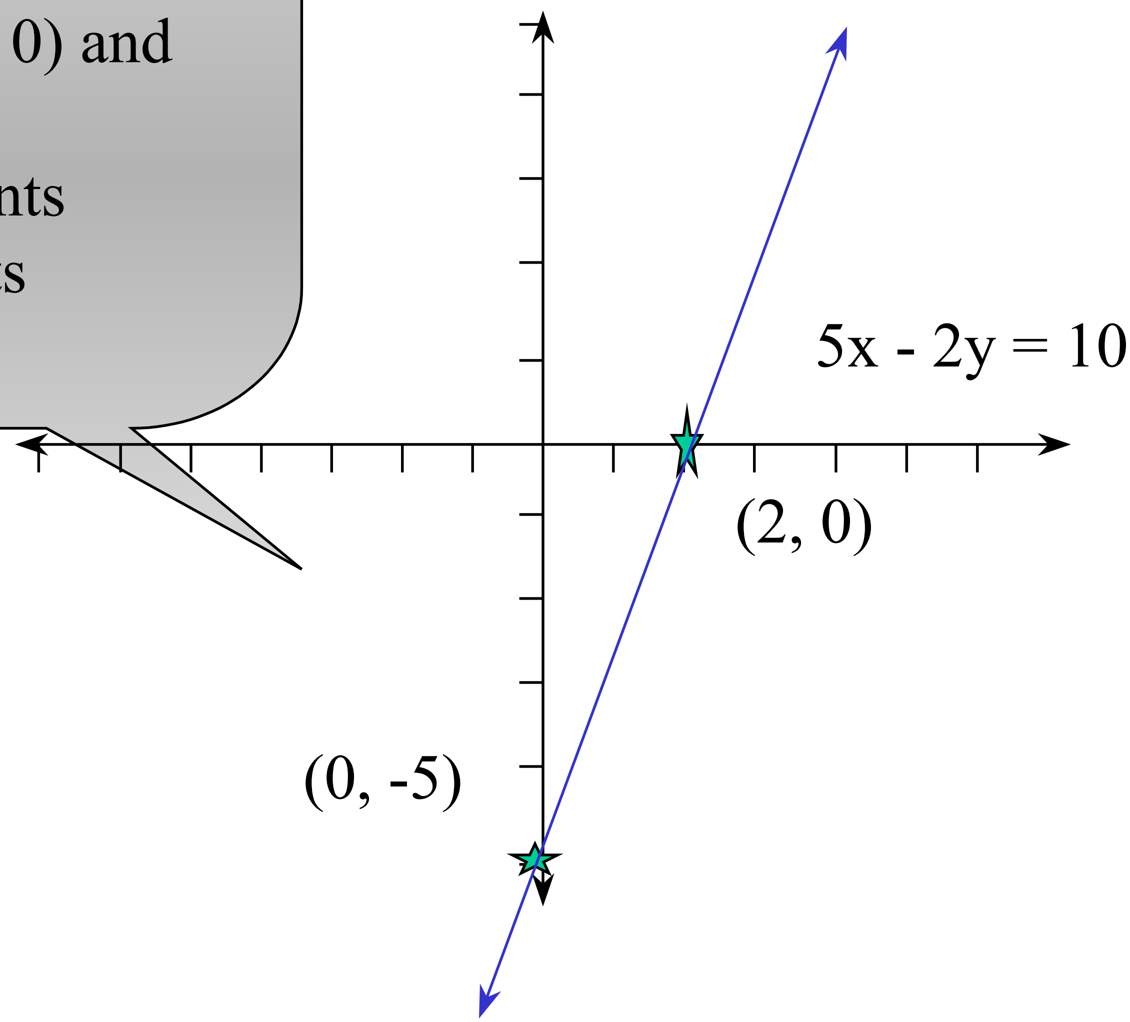
To Graph:

Step 1: Plot the points containing the x-intercept (2, 0) and y-intercept (0, -5)

Step 2: Label the two points

Step 3: Join the two points

Step 4. Label the graph



Your Turn

Procedure:

a) Determine the y-intercept by substituting zero for x in the equation and solving for one positive y.

b) Write the point as (0, b)

c) Determine the x-intercept by substituting zero for y in the equation and solving for one positive x.

d) Write the point as (x, 0)

e) Check calculation answers on Intercept slide.

f) Graph each equation and check your graphs on Graphic Answer slide.

a) $5x + 2y = 10$

b) $-3x + 4y = 12$

c) $2x - 3y = 24$

Intercept Slide

a) $5x + 2y = 10$

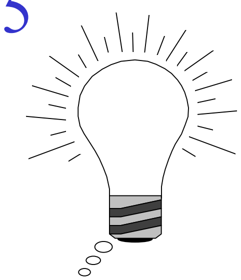
if $x = 0$

$5(0) + 2y = 10$

$2y = 10$

$y = 5$

Point $(0, 5)$



if $y = 0$

$5x + 2(0) = 10$

$5x = 10$

$x = 2$

Point $(2, 0)$

b) $-3x + 4y = 12$

if $x = 0$

$-3(0) + 4y = 12$

$4y = 12$

$y = 3$

Point $(0, 3)$

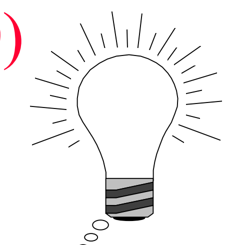
if $y = 0$

$-3x + 4(0) = 12$

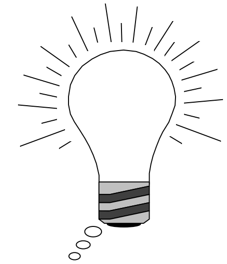
$-3x = 12$

$x = -4$

Point $(-4, 0)$



c) $2x - 3y = 24$



if $x = 0$

$2(0) - 3y = 24$

$-3y = 24$

$y = -8$

Point $(0, -8)$

if $y = 0$

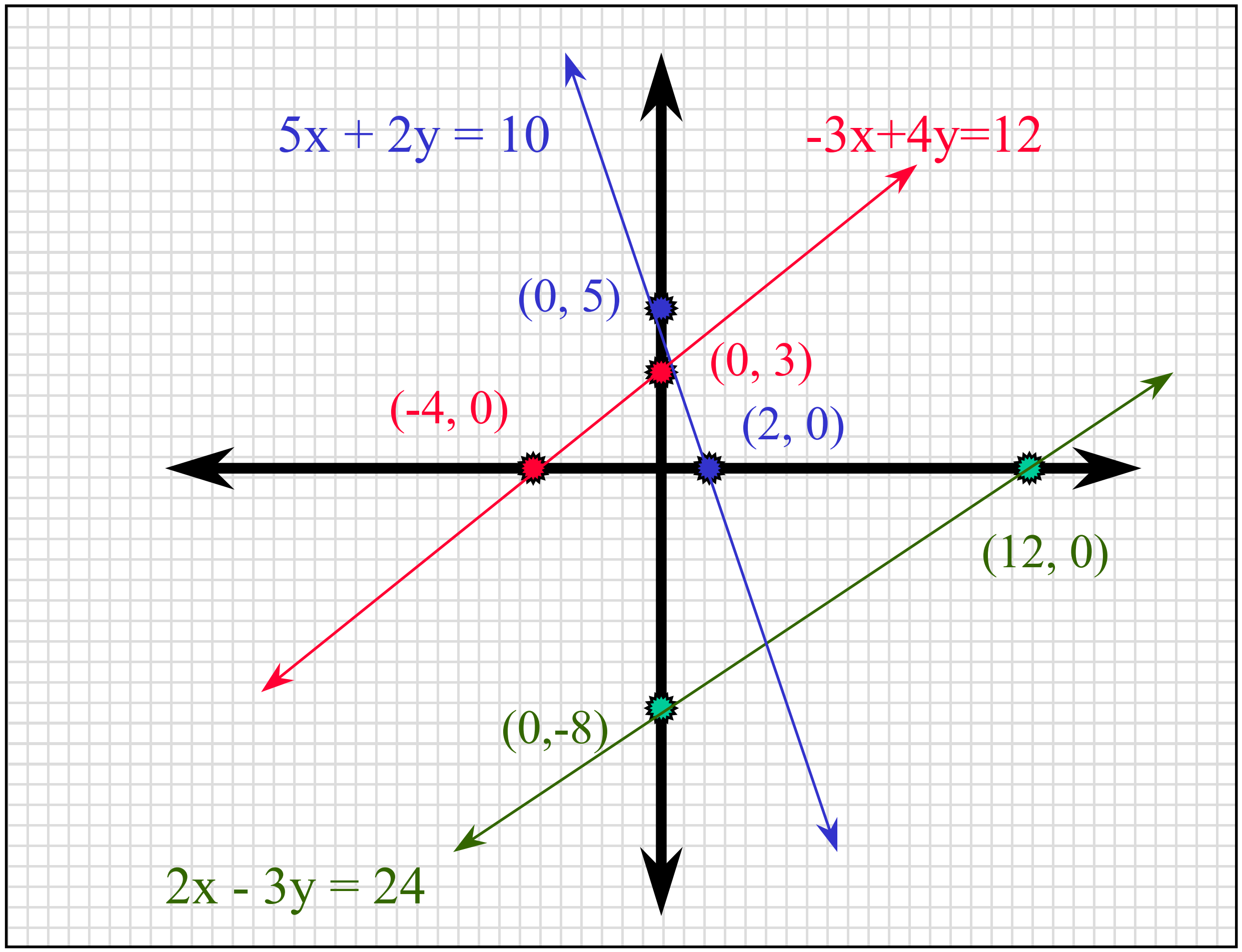
$2x - 3(0) = 24$

$2x = 24$

$x = 12$

Point $(12, 0)$

Graphic Answer



Characteristics of a Linear Equation or Linear Function

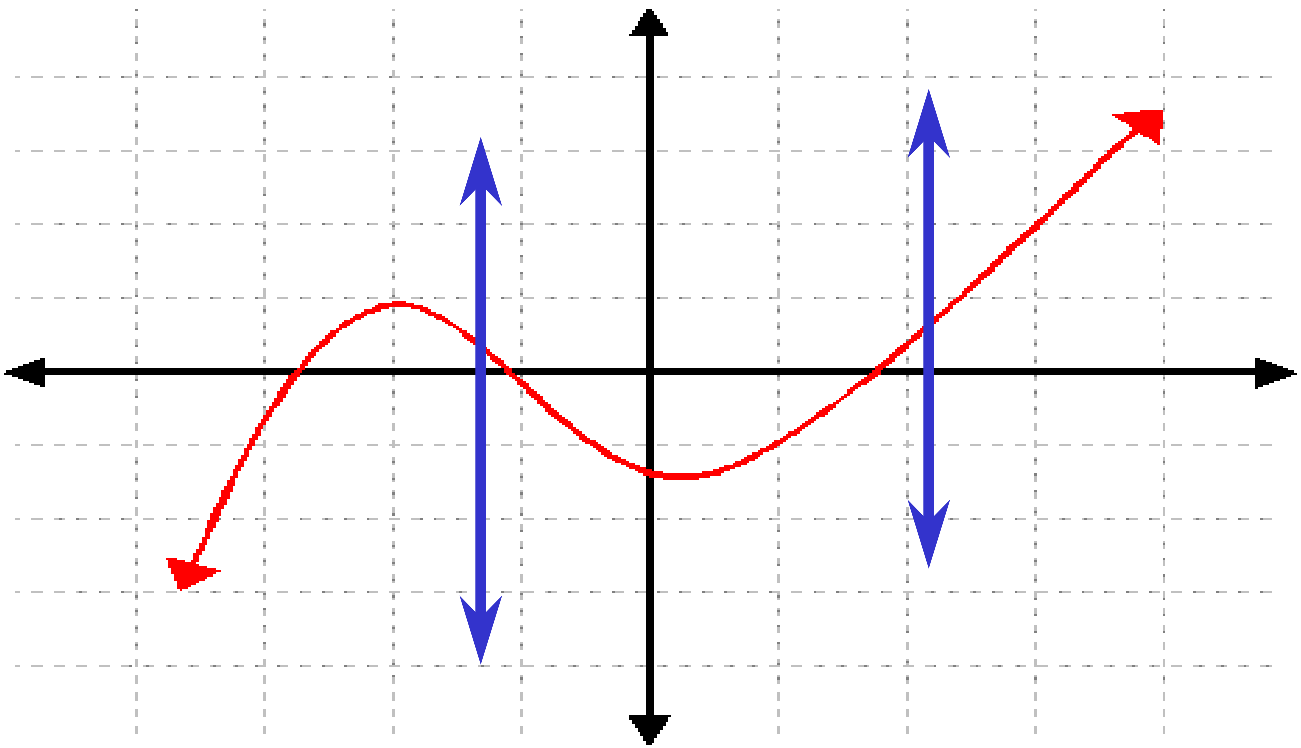
General Equation: **$Ax + By = C$**

1. x and y must be degree "1" - in other words have an exponent "1"
2. A, B, and C must be real numbers

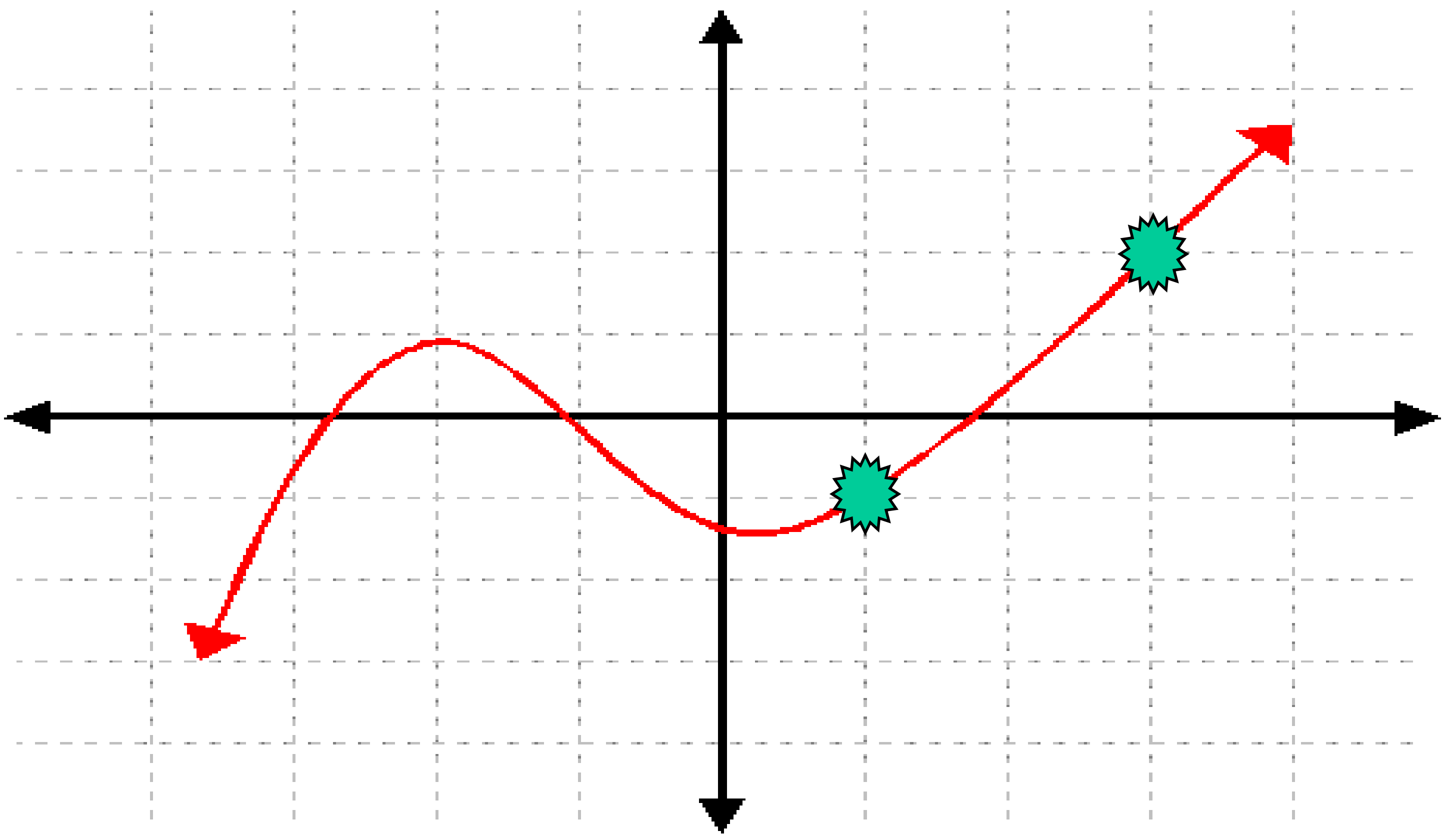
Two special cases:

1. when $A = 0$ the equation will read $By = C$
2. when $B = 0$ the equation will read $Ax = C$

A function is a relation (usually an equation) in which no two ordered pairs have the same x-coordinate when graphed. One way to tell if a graph is a function is the vertical line test, which says if it is possible for a vertical line to meet a graph more than once, the graph is not a function. The figure below is an example of a function



Functions are usually denoted by letters such as f or g. If the first coordinate of an ordered pair is represented by x, the second coordinate (the y coordinate) can be represented by f(x). In the figure below, f(1) = -1 and f(3) = 2.



V
E
R
HORIZONTAL
T
I
C
A
L

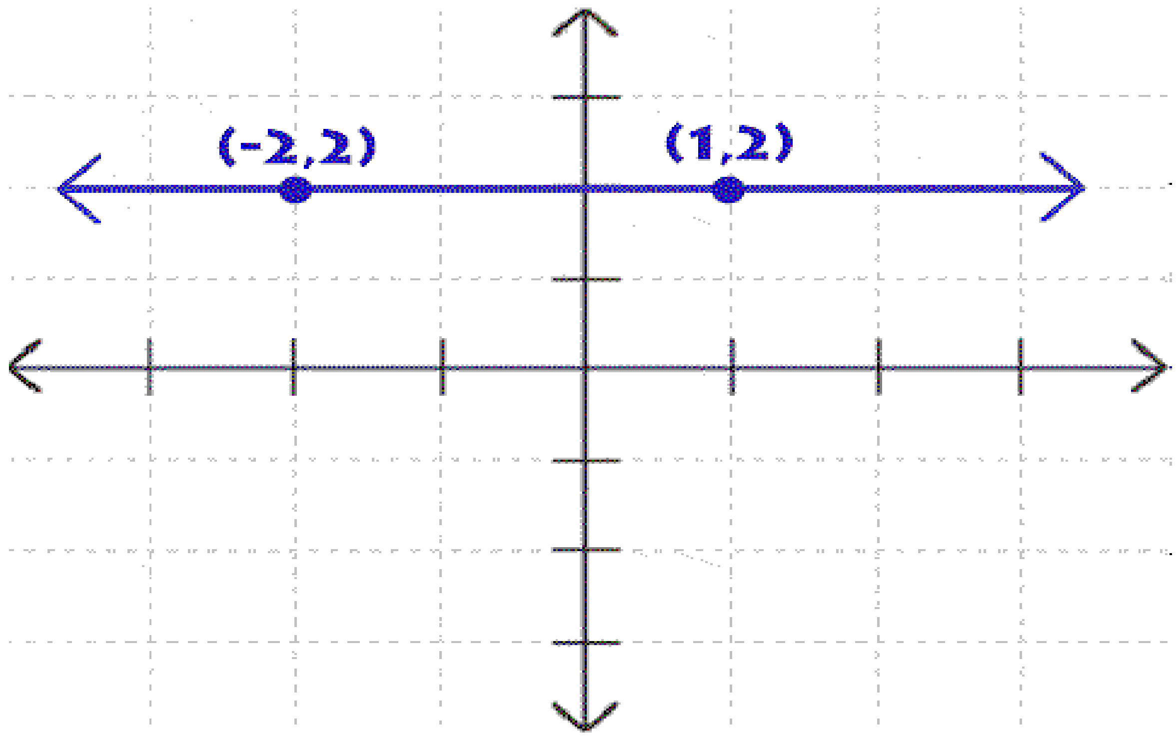
The most confusing types of lines are lines that are either **horizontal** or **vertical**. These are lines that are representative of an equation that has either an x variable or a y variable, but not both. An equation such as $y = 2$ says that no matter what you substitute for x, you always get $y = 2$. With this horizontal line, the slope is zero.

An equation such as $x = 4$ has two things to keep in mind. First of all it is vertical.line and for any vertical line, the slope is undefined, therefore we state that there is no slope. The other thing to remember is that no matter what you substitute in for y, you'll get $x =4$.

Example

1. Graph: $y = 2$

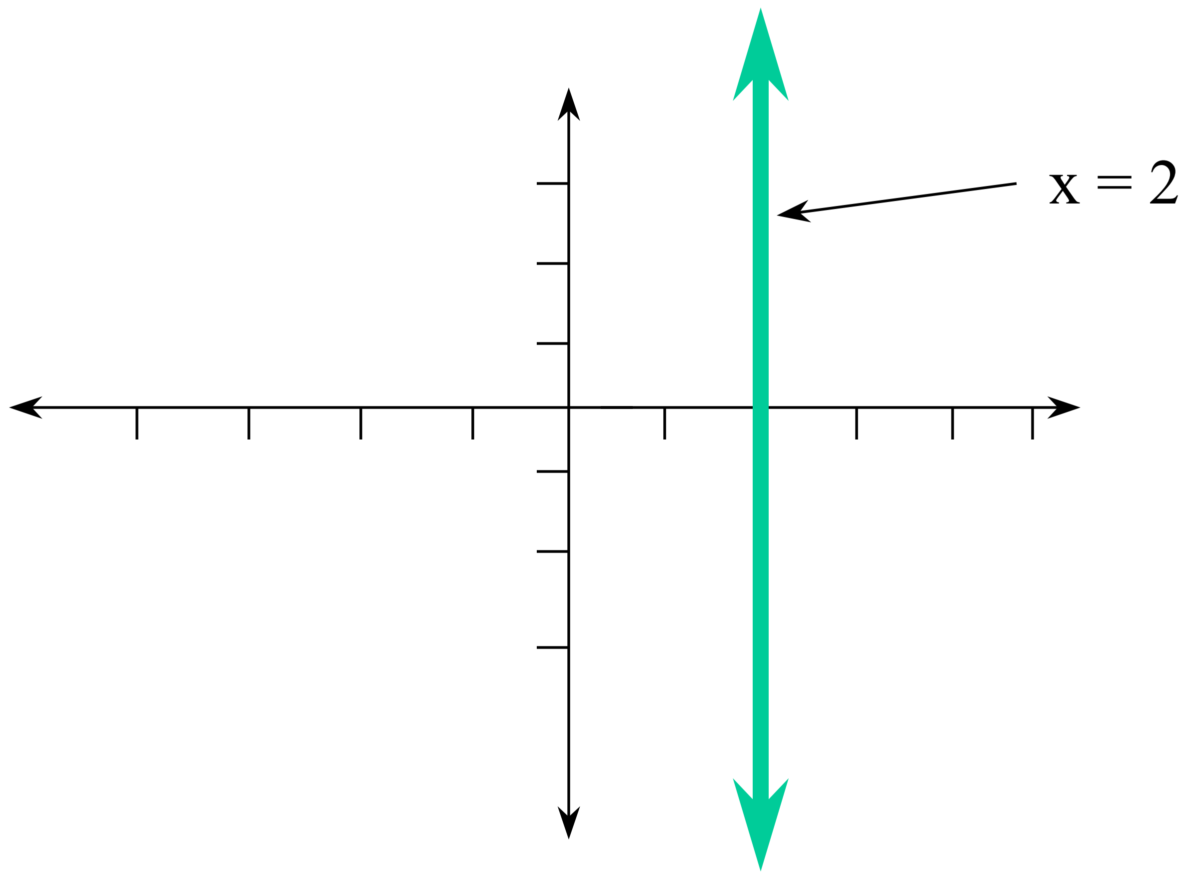
Solution: This equation indicates that all the y coordinates to be graphed are 2. Pick any two ordered pairs with 2 as the y coordinate and graph.



Example

2. Graph: $x = 2$

Solution: This equation indicates that all the x coordinates to be graphed are 2. Pick any two ordered pairs with 2 as the x coordinate and graph.



Your Turn

For each of the equations below

a) identify the type of line (vertical or horizontal)

b) draw the graphs of the equations

c) check your answers with those on the two next slides

a) $x = -4$

b) $y = -7$

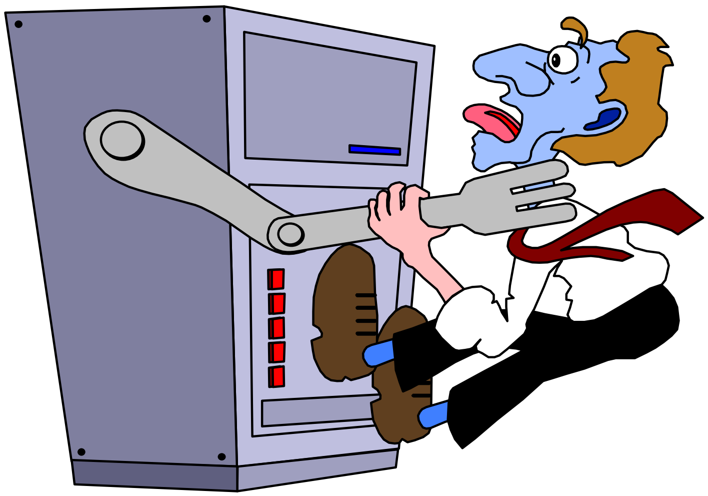
c) $x = 8$

d) $y = 5$

e) $3x = 12$

f) $-5y = 10$

g) $4y + 0x = 28$

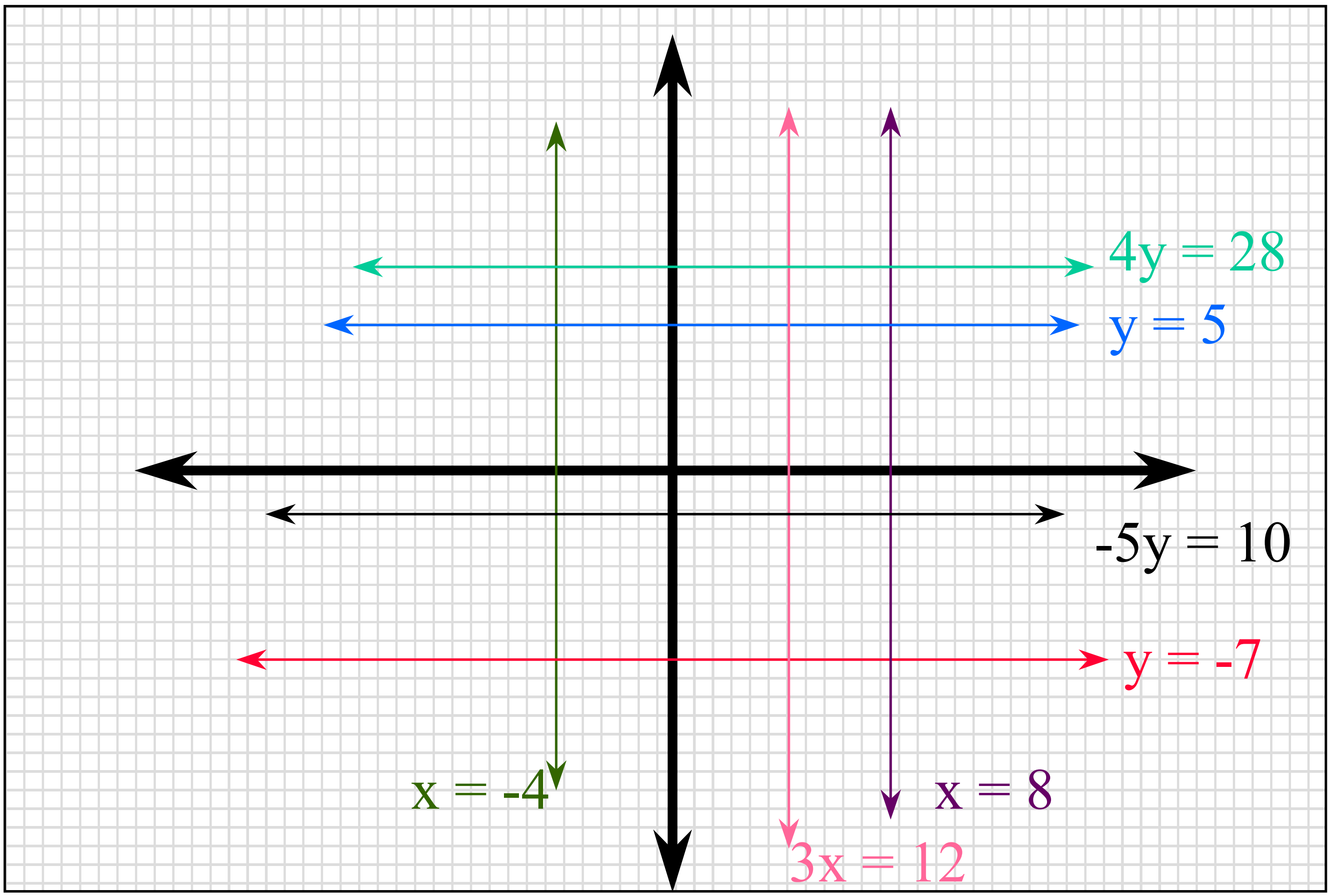


Types of lines defined by their equation

Questions	Type of Line
a) $x = -4$	a) vertical
b) $y = -7$	b) horizontal
c) $x = 8$	c) vertical
d) $y = 5$	d) horizontal
e) $3x = 12$	e) vertical
f) $-5y = 10$	f) horizontal
g) $4y + 0x = 28$	g) horizontal

remember to solve for one positive ‘y’

Graphic Solutions

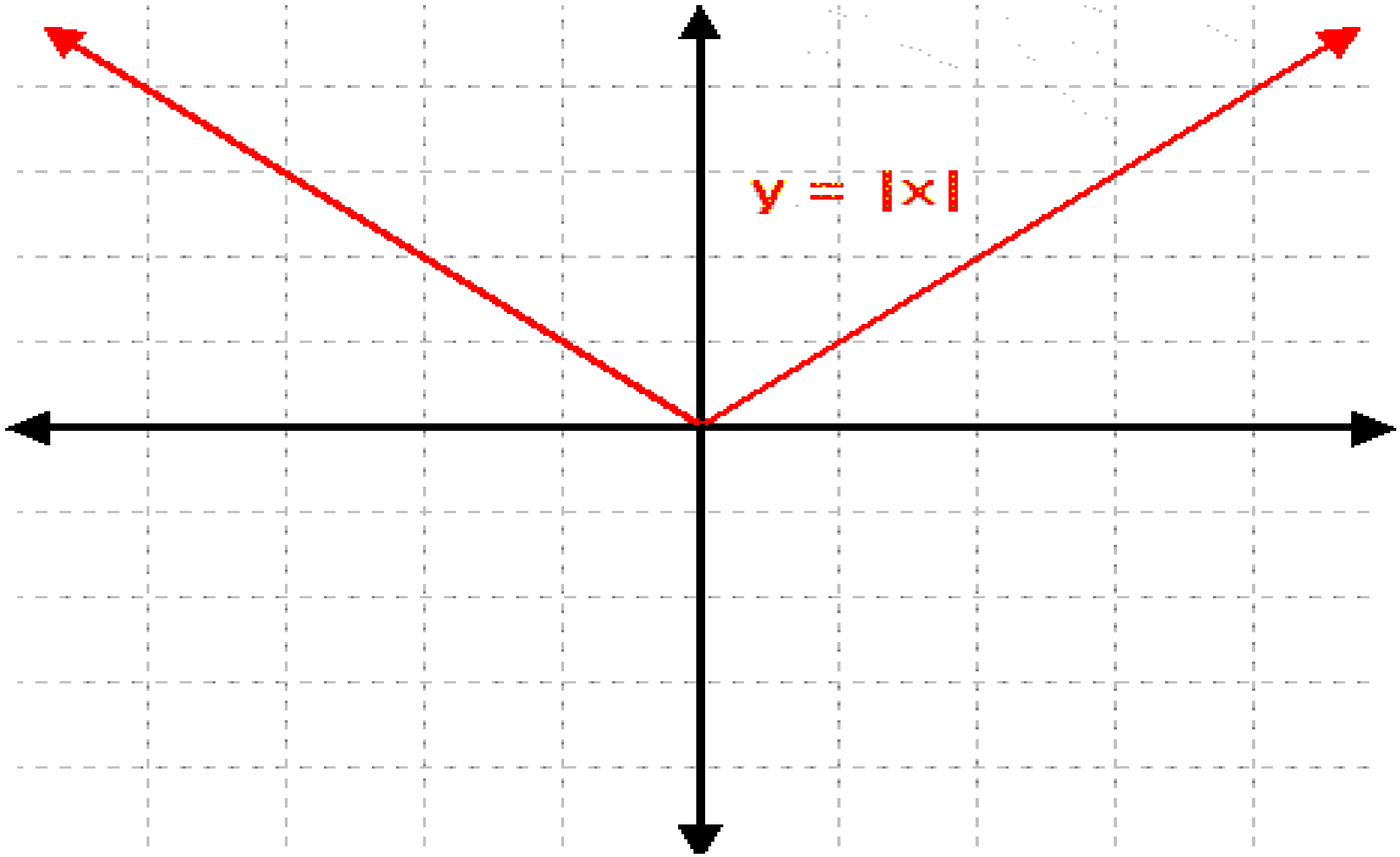


Other Special Lines

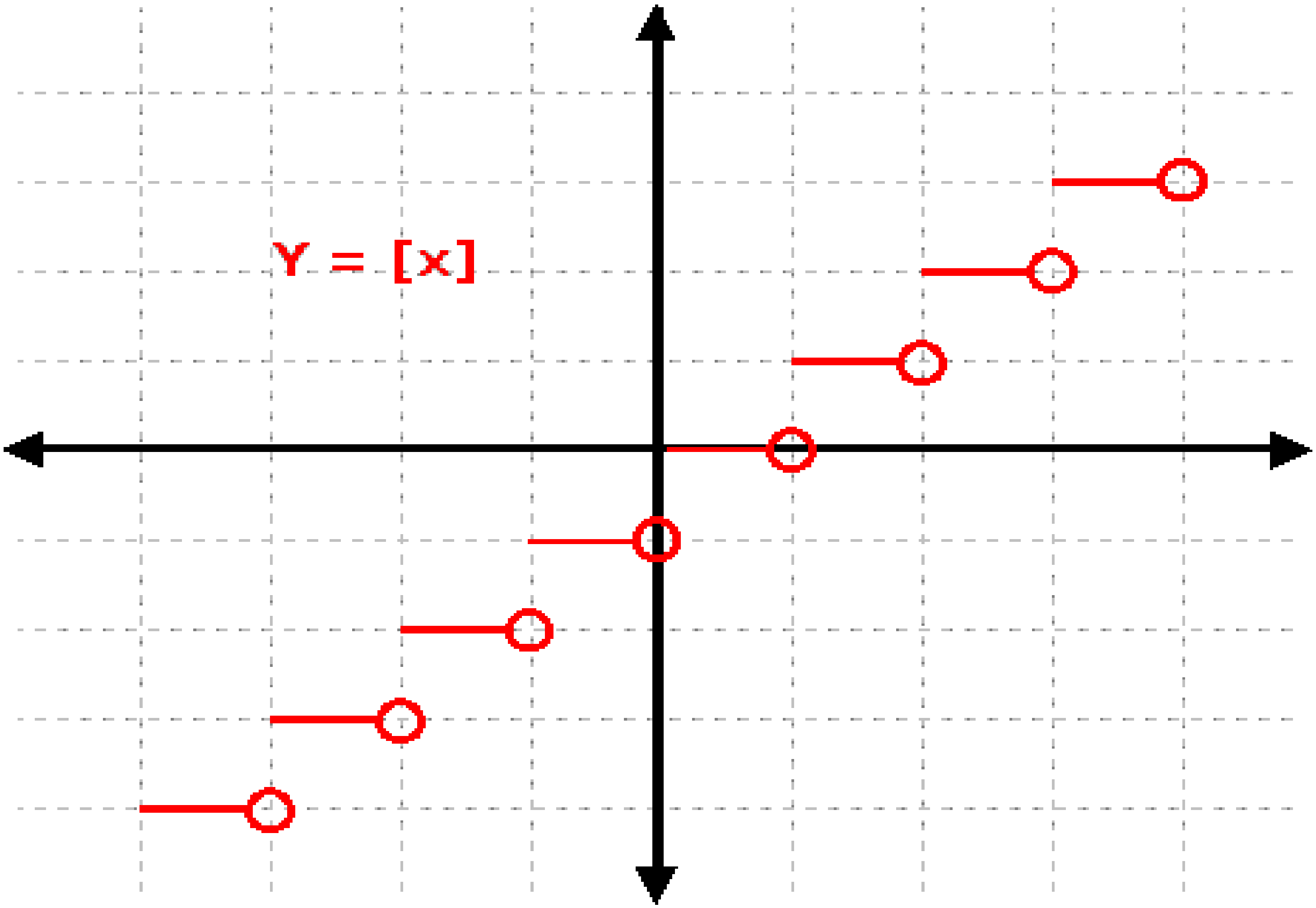
When a function is in the form of an equation, the domain is the set of numbers that are replacements for x that give a value for f(x) that is on the graph. Sometimes, certain replacements do not work, such as 0 in the following function: f(x) = 4/x (you cannot divide by 0). In that case, the domain is said to be x <> 0.

There are a couple of special functions whose graphs you should have memorized because they are sometimes hard to graph.

They are the absolute value function (below)



and the greatest integer function (below).



It is time to see what we have Learnt

Define the following terms:

a) coordinate plane
b) x-intercept
c) slope
d) y-intercept
e) origin
f) number line
g) ordered pair
h) axis
i) rise
j) run

Answers

a) coordinate plane -	a two dimensional plane marked by an x and y axis
b) x-intercept -	the value of x at the point where the line cuts the x-axis
c) slope -	the steepness of a line
d) y-intercept -	the value of y at the point where the line cuts the y-axis
e) origin -	the intersection point of the x and y axis and has coordinates (0,0)

f) line - an infinite number of points with a constant slope

g) ordered pair - a pair of numbers for which order is important (x, y)

h) axis - either a vertical or horizontal number line with a particular scale

i) rise - movement in a vertical direction (positive is considered up and negative is considered down)

j) run - movement horizontally and always considered right from the point

Graph the following equations using the indicated method:

a) $2x + y = 6$	(table of values)
b) $4x - 5y = 15$	(slope intercept method)
c) $3x - 6y = 18$	(intercept method)

a) $2x + y = 6$

$y = -2x + 6$

x	-5	3	5
y	16	0	-4

b) $4x - 5y = 15$

$-5y = -4x + 15$

$y = \dfrac{-4x}{-5} + \dfrac{15}{-5}$

$m = 4/5$

$b = -3$

point $(0, -3)$

c) $3x - 6y = 18$

if $x = 0$

$3(0) - 6y = 18$

$-6y = 18$

$y = -3$

Point $(0, -3)$

if $y = 0$

$3x - 6(0) = 18$

$3x = 18$

$x = 6$

Point $(6, 0)$

Graphic Solutions