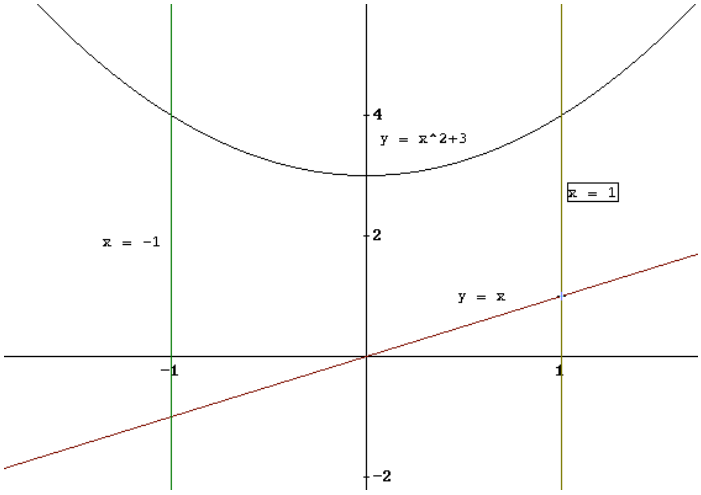
Areas Between The Curves

Sketch the region bounded by the given curves and find the area of the region:

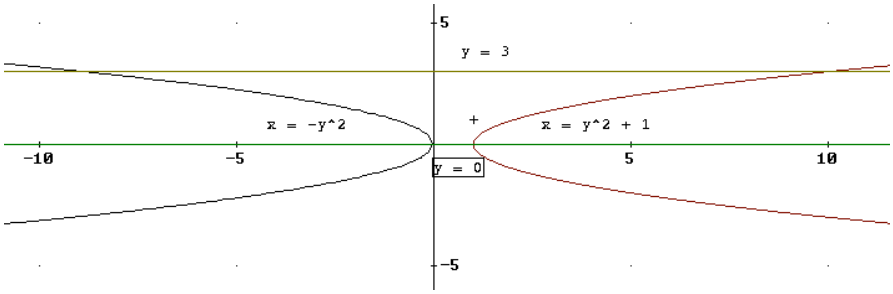
$f(x)=x^2=3, g(x)=x, x=-1, x=1$

1. $\int_{-1}^{1}\left[(x^2+3)-x\right]dx=\frac{x^3}{3}+3x-\frac{x^2}{2}\Big]_{-1}^{1}=\frac{20}{3}$



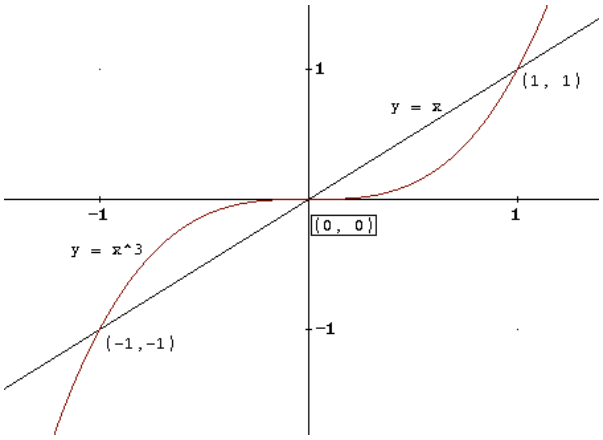
$x+y^2=0,\ x=y^2+1\ ,\ y=0,\ y=3$

2. $\int_0^3\left[\left(y^2+1\right)-\left(-y^2\right)\right]dy=\int_0^3\left[\left(2y^2+1\right)\right]dy=2\frac{y^3}{3}+y\Big]_0^3=21$



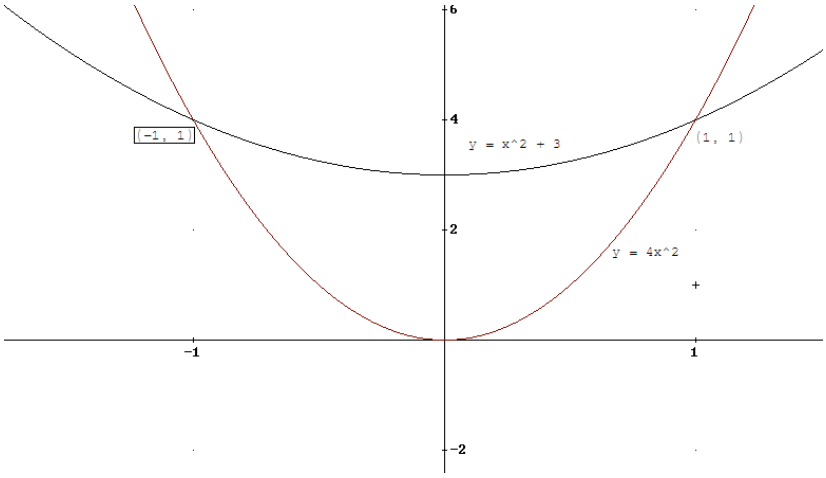
$y=x,\ y=x^3$

3. $\int_{-1}^{0}\left(x^3-x\right)dx+\int_0^1\left(x-x^3\right)dx=\frac{x^4}{4}-\frac{x^2}{2}\Big]_{-1}^{0}+\frac{x^2}{2}-\frac{x^4}{4}\Big]_0^1=\frac{1}{2}$



$y = 4x^2,\ y = x^2 + 3$

4. $\int_{-1}^{1}\left[\left(x^2+3\right)-4x^2\right]dx = \int_{-1}^{1}\left[\left(-3x^2+3\right)\right]dx = -3\frac{x^3}{3}+3x\Big]_{-1}^{1} = 4$

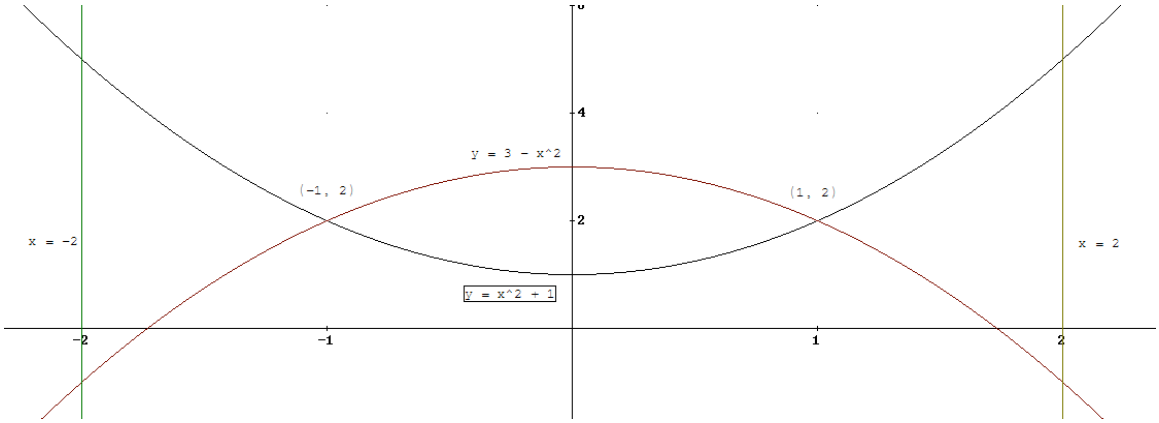


$y = x^2 + 1,\ y = 3 - x^2,\ x = -2,\ x = 2$

$\int_{-2}^{-1}\left[\left(x^2+1\right)-\left(3-x^2\right)\right]dx + \int_{-1}^{1}\left[\left(3-x^2\right)-\left(x^2+1\right)\right]dx + \int_{1}^{2}\left[\left(x^2+1\right)-\left(3-x^2\right)\right]dx =$

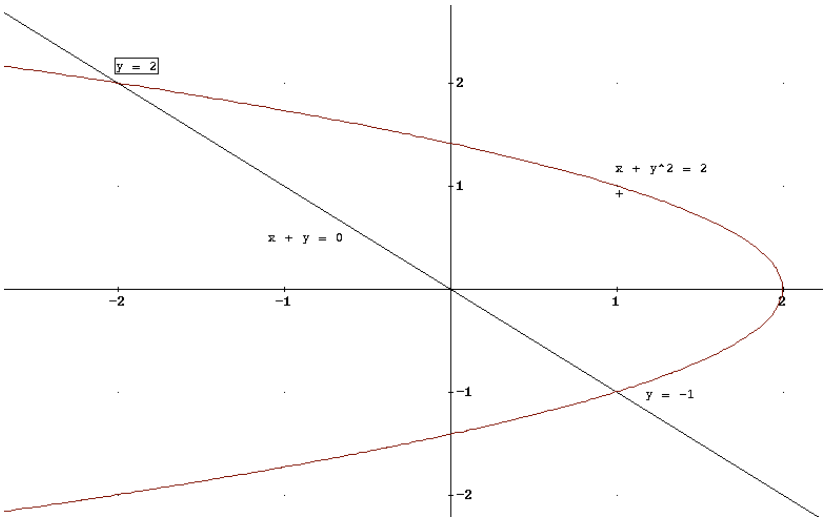
5. $\int_{-2}^{-1}\left[\left(2x^2-2\right)\right]dx + \int_{-1}^{1}\left[\left(2-2x^2\right)\right]dx + \int_{1}^{2}\left[\left(2x^2-2\right)\right]dx =$

$2\frac{x^3}{3}-2x\Big]_{-2}^{-1} + 2x - 2\frac{x^3}{3}\Big]_{-1}^{1} + 2\frac{x^3}{3}-2x\Big]_{1}^{2} = 8$



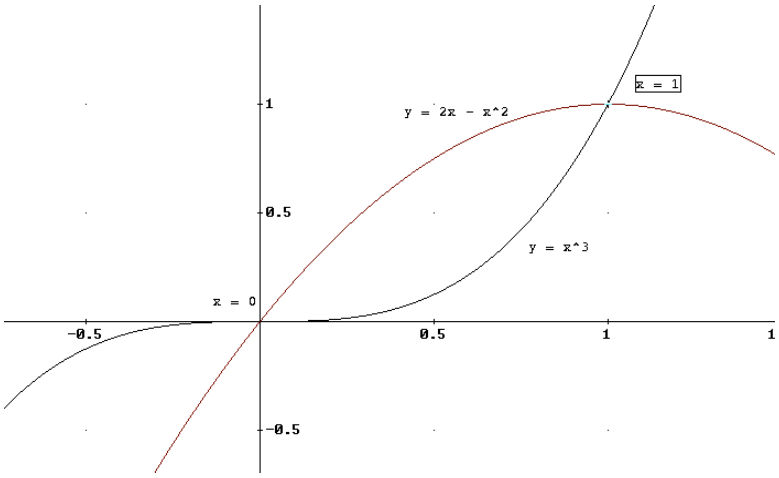
$x + y^2 = 2,\ x + y = 0$

6. $\int_{-1}^{2}\left[\left(2 - y^2\right) - (-y)\right]dy = 2y - \frac{y^3}{3} + \frac{y^2}{2}\Big]_{-1}^{2} = \frac{9}{2}$



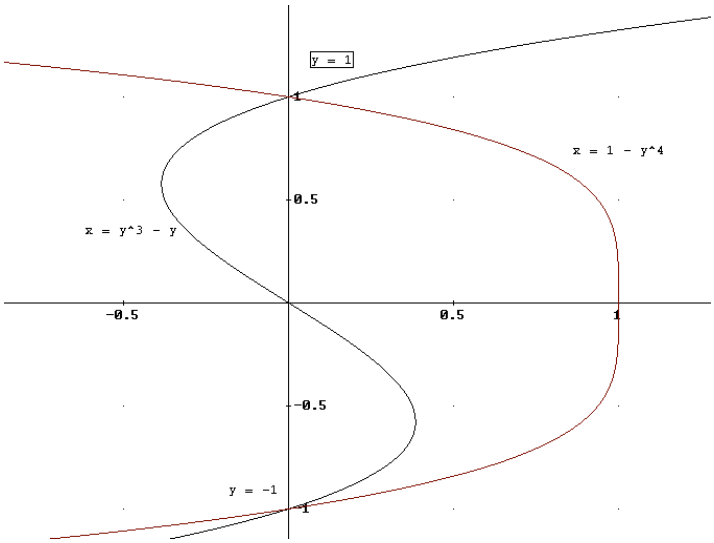
$y = 2x - x^2,\ y = x^3$

7. $\int_{0}^{1}\left[\left(2x - x^2\right) - x^3\right]dx = 2\frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4}\Big]_{0}^{1} = \frac{5}{12}$



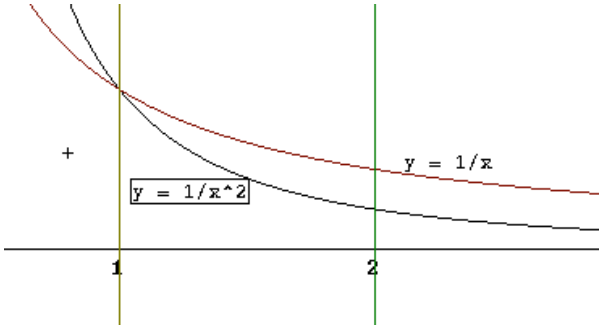
$x = 1 - y^4,\ x = y^3 - y$

8. $\int_{-1}^{1}\left[\left(1-y^4\right)-\left(y^3-y\right)\right]dy = y - \frac{y^5}{5} - \frac{y^4}{4} - \frac{y^2}{2}\Big]_{-1}^{1} = \frac{8}{5}$



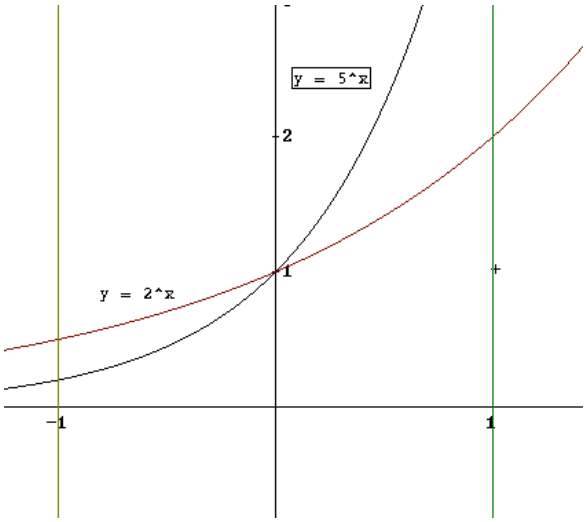
$y = 1/x,\ y = 1/x^2,\ x = 1,\ x = 2$

9. $\int_{1}^{2}\left[\left(1/x\right)-\left(1/x^2\right)\right]dx = \int_{1}^{2}\left[x^{-1}-x^{-2}\right]dx = \ln x - \frac{x^{-1}}{-1}\Big]_{1}^{2} = 0.1931$



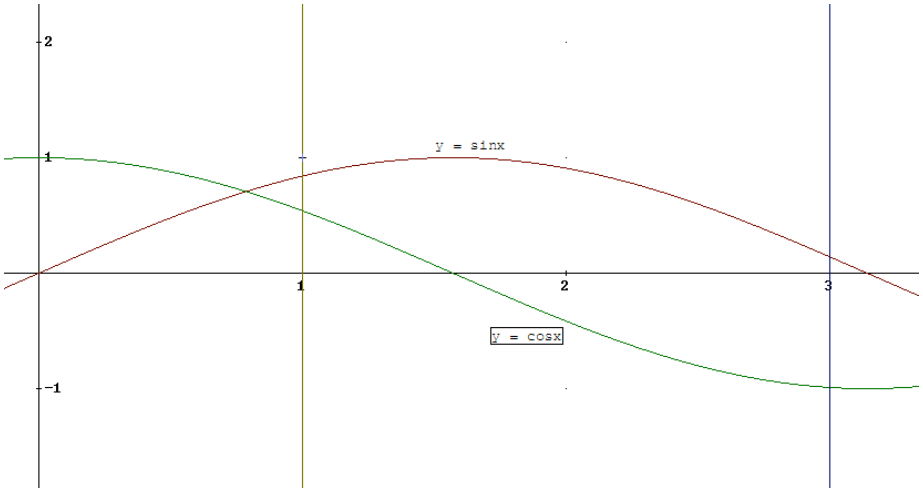
$y = 2^x,\ y = 5^x,\ x = -1,\ x = 1$

10. $\int_{-1}^{0}\left[2^x - 5^x\right]dx + \int_{0}^{1}\left[5^x - 2^x\right]dx = \left[\frac{2^x}{\ln 2} - \frac{5^x}{\ln 5}\right]_{-1}^{0} + \left[\frac{5^x}{\ln 5} - \frac{2^x}{\ln 2}\right]_{0}^{1} = 1.26$



$y = \sin\ x,\ y = \cos\ x,\ x = 1,\ x = 3$

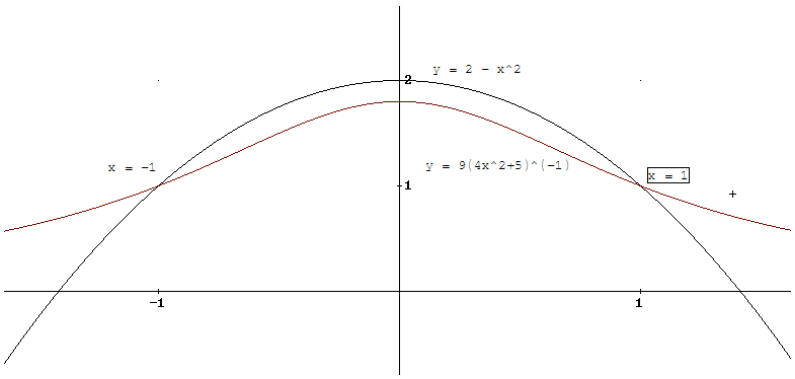
11. $\int_{1}^{3}(\sin x - \cos x)dx = -\cos x - \sin x\Big]_{1}^{3} = 2.23$



12.

$y = 9(4x^2 + 5)^{-1},\ y = 2 - x^2$

$$\int_{-1}^{1}\left[\left(2-x^2\right)-\left(9(4x^2+5)^{-1}\right)\right]dx = 2x - \frac{x^3}{3} - 9\frac{1}{\frac{\sqrt{5}}{2}}\tan^{-1}\frac{x}{\frac{\sqrt{5}}{2}}\Bigg|_{-1}^{1} = 2x - \frac{x^3}{3} - 9\frac{2}{\sqrt{5}}\tan^{-1}\frac{2}{\sqrt{5}}\Bigg]_{-1}^{1} = 0.396$$



Answer = 0.396