Partial Fractions

Points to note:

1. If denominator had linear factors of the from $x, (x+1), (2x-3)$ the starting rational expression would be $\dfrac{A}{x} + \dfrac{B}{(x+1)} + \dfrac{C}{(2x-3)}$.

2. If the denominator had repeating linear factors of the form $x^3, (2x-1)^2$ the starting rational expression would be $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x^3} + \dfrac{D}{(2x-1)} + \dfrac{E}{(2x-1)^2}$

3. If the denominator had quadratic factors of the form $(x^2+2), (x^2+x-1)$ the starting rational expression would be $\dfrac{Ax+B}{(x^2+2)} + \dfrac{Cx+D}{(x^2+x-1)}$

4. If the denominator had quadratic factors of the form $(x^2+1)^2(2x^2+5x+1)^2$ the starting rational expression would be
$$\dfrac{Ax+B}{(x^2+1)} + \dfrac{Cx+D}{(x^2+1)^2} + \dfrac{Ex+F}{(2x^2+5x+1)} + \dfrac{Gx+H}{(2x^2+5x+1)^2}$$

5. If the denominator had a mixture of linear and quadratic some single and some repeating $x^3, (3x+1), (2x^2+1)^2, (5x^2+x+1)$ the starting rational expression would be $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x^3} + \dfrac{D}{(3x+1)} + \dfrac{Ex+F}{(2x^2+1)} + \dfrac{Gx+H}{(2x^2+1)^2} + \dfrac{Ix+J}{(5x^2+x+1)}$

Assignment:

1. $\displaystyle\int \dfrac{x+2}{x^2-2x+1}\,dx$

2. $\displaystyle\int \dfrac{1}{x^3+x}\,dx$

3. $\displaystyle\int \dfrac{x}{(x-4)^2}\,dx$

4. $\displaystyle\int \dfrac{x^2+3x-2}{x^3+5x}\,dx$

5. $\displaystyle\int \dfrac{x^3}{\left(x^2+2\right)^2}\,dx$

6. $\displaystyle\int \dfrac{x^2+6x+4}{x^4+5x^2+4}\,dx$

7. $\displaystyle\int \dfrac{x}{x^4+7x^2+6}\,dx$

8. $\displaystyle\int \dfrac{x^4-x^3-x-1}{x^3-x^2}\,dx$