Integration

1. $\int (3x^{\frac{2}{3}} + \frac{8}{x})dx = 3\int x^{\frac{2}{3}}dx + 8\int x^{-1}dx = 3 \cdot \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + 8\ln x = \frac{9}{5}x^{\frac{5}{3}} + 8\ln x + c$

2. $\int (x^4 - 3\sqrt{x})dx = \int x^4 dx - 3\int x^{\frac{1}{2}}dx = \frac{x^5}{5} - 3 \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} = \frac{x^5}{5} - 2x^{\frac{3}{2}} + c$

3. $\int \sqrt[4]{x}\,dx = \int x^{\frac{1}{4}}dx = \frac{x^{\frac{5}{4}}}{\frac{5}{4}} = \frac{4x^{\frac{5}{4}}}{5} + c$

4. $\int \frac{1}{x^2}dx = \int x^{-2}dx = \frac{x^{-1}}{-1} = -x^{-1} + c$

5. $\int (x^3 + 2x + \sin x)dx = \int x^3 dx + 2\int xdx + \int \sin x dx = \frac{x^4}{4} + 2\frac{x^2}{2} + (-\cos x) = \frac{x^4}{4} + x^2 - \cos x + c$

6. $\int \sin(3x)dx = \frac{1}{3}\int \sin(u)du = \frac{1}{3}(-\cos u) = -\frac{1}{3}\cos(3x) + c$

   $u = 3x \Rightarrow du = 3dx \Rightarrow \frac{1}{3}du = dx$

7. $\int 2^x dx = \frac{1}{\ln 2}2^x + c$

8. $\int e^{2x}dx = \frac{1}{2}\int e^u du = \frac{1}{2} \cdot e^u = \frac{1}{2} \cdot e^{2x} + c$

   $u = 2x \Rightarrow du = 2dx \Rightarrow \frac{1}{2}du = dx$

9. $\int (3e^x + 2\sqrt[5]{x})dx = 3\int e^x dx + 2\int x^{\frac{1}{5}}dx = 3e^x + 2\frac{x^{\frac{6}{5}}}{\frac{6}{5}} = 3e^x + \frac{5}{3}x^{\frac{6}{5}} + c$

10. $\int (3\sin x + 4\cos x)dx = 3\int \sin x dx + 4\int \cos x dx = 3(-\cos x) + 4(\sin x) = -3\cos x + 4\sin x + c$

11. $\int\left(\dfrac{1}{x^3}+\dfrac{2}{x^5}\right)dx=\int x^{-3}dx+2\int x^{-5}dx=\dfrac{x^{-2}}{-2}+2\dfrac{x^{-4}}{-4}=-\dfrac{x^{-2}}{2}-\dfrac{x^{-4}}{2}+c$

12. $\int\left(\dfrac{1}{x^2}\right)(1+2x+3x^2+4x^3)dx=\int x^{-2}dx+2\int x^{-1}dx+3\int dx+4\int x\,dx=\dfrac{x^{-1}}{-1}+2\ln x+3x+4\dfrac{x^2}{2}=$

$-x^{-1}+2\ln x+3x+2x^2+c$

13. $\int 2x\sqrt{x^2+1}\,dx=\int 2x(x^2+1)^{\frac{1}{2}}dx=\int u^{\frac{1}{2}}du=\dfrac{u^{\frac{3}{2}}}{\frac{3}{2}}=\dfrac{2}{3}u^{\frac{3}{2}}=\dfrac{2}{3}(x^2+1)^{\frac{3}{2}}+c$

$u=x^2+1\Rightarrow du=2xdx$

14. $\int \cos^2 x\sin x\,dx=\int(\cos x)^2\sin x\,dx=-\int u^2du=-\dfrac{u^3}{3}=-\dfrac{1}{3}(\cos x)^3+c$

$u=\cos x\Rightarrow du=-\sin xdx\Rightarrow -du=\sin xdx\Rightarrow$

15. $\int\dfrac{e^x}{1+e^{2x}}dx=\int\dfrac{1}{1+u^2}du=\int\dfrac{1}{1^2+u^2}du=\dfrac{1}{1}\tan^{-1}\dfrac{u}{1}=\tan^{-1}(e^x)+c$

$u=e^x\Rightarrow du=e^xdx$

16. $\int 2x(x^2+4)^{\frac{3}{2}}dx=\int u^{\frac{3}{2}}du=\dfrac{u^{\frac{5}{2}}}{\frac{5}{2}}=\dfrac{2}{5}u^{\frac{5}{2}}=\dfrac{2}{5}(x^2+4)^{\frac{5}{2}}+c$

$u=x^2+4\Rightarrow du=2xdx$

17. $\int(x+1)(x^2+2x-4)dx=\dfrac{1}{2}\int u\,du=\dfrac{1}{2}\dfrac{u^2}{2}=\dfrac{u^2}{4}=\dfrac{(x^2+2x-4)^2}{4}+c$

$u=(x^2+2x-4)\Rightarrow du=(2x+2)dx\Rightarrow du=2(x+1)dx\Rightarrow\dfrac{1}{2}du=(x+1)dx$

$\int\dfrac{\sec^2 x}{\tan^3 x}dx=\int\dfrac{\frac{1}{\cos^2 x}}{\frac{\sin^3 x}{\cos^3 x}}dx=\int\dfrac{1\cdot\cos^3 x}{\cos^2 x\cdot\sin^3 x}dx=\int\dfrac{\cos x}{\sin^3 x}dx$

18. $\int\dfrac{1}{u^3}du=\int u^{-3}du=\dfrac{u^{-2}}{-2}=-\dfrac{1}{2}(\sin x)^{-2}+c$

$u=\sin x\Rightarrow du=\cos xdx$

20. $$\int (x+1)(\cos(x^2+2x)dx = \frac{1}{2}\int \cos u\,du = \frac{1}{2}\sin u = \frac{1}{2}\sin\left(x^2+2x\right)+c$$

$$u = \left(x^2+2x\right) \Rightarrow du = (2x+2)dx \Rightarrow du = 2(x+1)dx \Rightarrow \frac{1}{2}du = (x+1)dx$$

22. $$\int \frac{x^{\frac{1}{3}}}{(x^{\frac{4}{3}}+1)^2}dx = \frac{3}{4}\int \frac{1}{u^2}du = \frac{3}{4}\int u^{-\frac{3}{2}}du = \frac{3}{4}\cdot\frac{u^{-\frac{1}{2}}}{-1} = -\frac{3}{2}u^{-\frac{1}{2}} = -\frac{3}{2}\left(x^{\frac{4}{3}}+1\right)^{-\frac{1}{2}}+c$$

$$u = x^{\frac{4}{3}}+1 \Rightarrow du = \frac{4}{3}x^{\frac{1}{3}}dx \Rightarrow \frac{3}{4}du = x^{\frac{1}{3}}dx$$

24. $$\int e^{\sin x}\cos x dx = \int e^u du = e^u = e^{\sin x}+c$$

$$u = \sin x \Rightarrow du = \cos xdx$$

26. $$\int \frac{e^{\frac{1}{x}}}{x^2}dx = -\int e^u du = -e^u = -e^{\frac{1}{x}}+c$$

$$u = \frac{1}{x} \Rightarrow u = x^{-1} \Rightarrow du = -1x^{-2}dx \Rightarrow -du = \frac{1}{x^2}dx$$

$$\int \frac{x+1}{\sqrt{x^2+2x+3}}dx = \int \frac{x+1}{\left(x^2+2x+3\right)^{\frac{1}{2}}}dx = \int (x+1)\left(x^2+2x+3\right)^{-\frac{1}{2}}dx =$$

27. $\dfrac{1}{2}\int u^{-\frac{1}{2}}du = \dfrac{1}{2}\cdot\dfrac{u^{\frac{1}{2}}}{\frac{1}{2}} = \left(x^2+2x+3\right)^{\frac{1}{2}}+c$

$u = \left(x^2+2x+3\right)\Rightarrow du = (2x+2)dx \Rightarrow du = 2(x+1)dx \Rightarrow \dfrac{1}{2}du = (x+1)dx$

28. $\int \dfrac{x^3}{1+x^4}dx = \dfrac{1}{4}\int\dfrac{1}{u}du = \dfrac{1}{4}\int u^{-1}du = \dfrac{1}{4}\ln u = \dfrac{1}{4}\ln\left(1+x^4\right)+c$

$u = 1+x^4 \Rightarrow du = 4x^3dx \Rightarrow \dfrac{1}{4}du = x^3dx$

29. $\int \dfrac{1}{x\ln x}dx = \int\dfrac{1}{u}du = \int u^{-1}du = \ln u = \ln(\ln x)+c$

$u = \ln x \Rightarrow du = \dfrac{1}{x}dx$

30. $\int \dfrac{x^3+x-1}{x^2+1}dx = \int x - \dfrac{1}{x^2+1}dx = \int xdx - \int\dfrac{1}{x^2+1}dx = \dfrac{x^2}{2} - \dfrac{1}{1}\tan^{-1}\dfrac{x}{1} = \dfrac{x^2}{2} - \tan^{-1}x+c$

31. $\int \dfrac{\cos x}{1+\sin x}dx = \int\dfrac{1}{u}du = \int u^{-1}du = \ln u = \ln(1+\sin x)+c$

$u = 1+\sin x \Rightarrow du = \cos xdx$

32. $\int \dfrac{\sin(\ln x)}{x}dx = \int \sin udx = -\cos udu = -\cos(\ln(x))+c$

$u = \ln x \Rightarrow du = \dfrac{1}{x}dx$

33. $\int \dfrac{e^{2x}}{(1+e^{2x})}dx = \dfrac{1}{2}\int\dfrac{1}{u}du = \dfrac{1}{2}\ln u = \dfrac{1}{2}\ln\left(1+e^{2x}\right)+c$

$u = \left(1+e^{2x}\right)\Rightarrow du = 2e^{2x}dx \Rightarrow \dfrac{1}{2}du = e^{2x}dx$

34.

$$\int \frac{\sqrt{1+\ln x}}{x}dx = \int \frac{(1+\ln x)^{\frac{1}{2}}}{x}dx = \int u^{\frac{1}{2}}du = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3}(1+\ln x)^{\frac{3}{2}} + c$$

$$u = 1 + \ln x \Rightarrow du = \frac{1}{x}dx$$