1. $\int x^4\,dx = \frac{x^5}{5}$ 2. $\int e^x\,dx = e^x$ 3. $\int \sin x\,dx = \cos x$

4. $\int 5^x\,dx = \frac{1}{\ln 5}\cdot 5^x$ 5. $\int \cos x\,dx = \sin x$ 6. $\int \frac{1}{x}\,dx = \ln x$

7. $\int x^{-5}\,dx = \frac{x^{-4}}{-4}$ 8. $\int \frac{2}{x+1}\,dx = 2\int \frac{1}{x+1}\,dx = 2\ln x$.

9. $\int \cos 3x\,dx = \frac{1}{3}\int \cos u\,du = \frac{1}{3}\sin u = \frac{1}{3}\sin(3x)$

$u = 3x$

$du = 3\,dx$ $\frac{1}{3}du = dx$

10. $\int 7^{3x}\,dx = \frac{1}{3}\int 7^u\,du = \frac{1}{3\ln 7}\cdot 7^u = \frac{1}{3\ln 7}\cdot 7^{3x}$

$u = 3x$

$du = 3\,dx$

$\frac{1}{3}du = dx$

11. $\int x^2(x-3)\,dx = \int x^3 - 3x^2\,dx = \int x^3\,dx - 3\int x^2\,dx = \frac{x^4}{4} - \frac{3x^3}{3}$

12. $\int \frac{1}{x^5}\,dx = \int x^{-5}\,dx = \frac{x^{-4}}{-4}$

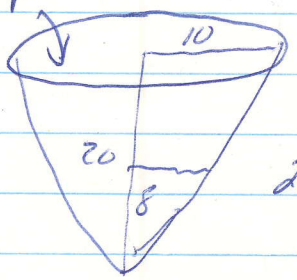
13. $\int x^4 - \frac{2}{x^3}\,dx = \int x^4\,dx - 2\int x^{-3}\,dx = \frac{x^5}{5} - \frac{2x^{-2}}{-2} = \frac{x^5}{5} + x^{-2}$

14. $\int 2x^3 - 5x + 1\,dx = \frac{2x^4}{4} - \frac{5x^2}{2} + x$

1. $y^2 - 2x^2 = 2 \quad 2y\frac{dy}{dt} - 4x\frac{dx}{dt} = 0 \quad 2(-2)\frac{dy}{dt} - 4(1)\left(\frac{1}{2}\right) = 0$

$-4\frac{dy}{dt} - 2 = 0 \quad -4\frac{dy}{dt} = 2 \quad \frac{dy}{dt} = -\frac{1}{2}$

2. $5\text{ ft}^3/\text{min}$



$\frac{r}{h} = \frac{10}{20} \quad 20r = 10h \quad r = \frac{1}{2}h$

$V = \frac{1}{3}\pi r^2 h \quad V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h \quad V = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{12}\pi 3h^2\frac{dh}{dt} \quad 5 = \frac{1}{4}\pi(8)^2\frac{dh}{dt}$

$5 = 16\pi\frac{dh}{dt}$

$\frac{5}{16\pi} = \frac{dh}{dt}$

3. $V = \frac{4}{3}\pi r^3$

Need to know $d = 2r$ or $r = d/2 \quad V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \quad V = \frac{\pi}{6}d^3$

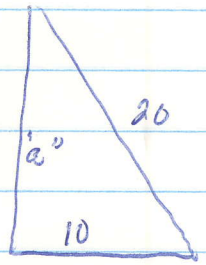
Need to know diameter after 5 minutes ($15 \times 5 = 75$ cubic ft of air)

$V = \frac{\pi}{6}d^3 \quad 75 = \frac{\pi}{6}d^3 \quad \frac{450}{\pi} = d^3 \quad \sqrt[3]{\frac{450}{\pi}} = d$

$\frac{dV}{dt} = \frac{\pi}{6}\cdot 3d^2\frac{dd}{dt} \quad \frac{dV}{dt} = \frac{\pi}{2}\left(\sqrt[3]{\frac{450}{\pi}}\right)^2\frac{dd}{dt} \quad 15 = \frac{\pi}{2}\left(\sqrt[3]{\frac{450}{\pi}}\right)^2\frac{dd}{dt}$

$\frac{30}{\pi} = \left(\sqrt[3]{\frac{450}{\pi}}\right)^2\frac{dd}{dt} \quad \frac{30}{\pi\sqrt[3]{\frac{450}{\pi}}} = \frac{dd}{dt}$

4. $\frac{da}{dt} = 2$ ft/sec



$c^2 = a^2 + b^2$

$20^2 = a^2 + 10^2$

$300 = a^2$

$10\sqrt{3} = a$

$2c\frac{dc}{dt} = 2a\frac{da}{dt} + 2b\frac{db}{dt}$

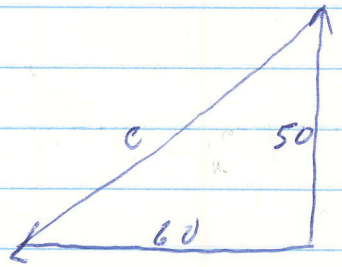
$0 = 10\sqrt{3}\left(\frac{da}{dt}\right) + 10(3)$

$\rightarrow \frac{db}{dt} = 3$ ft/sec

$0 = 10\sqrt{3}\left(\frac{da}{dt}\right) + 30$

$-30 = 10\sqrt{3}\frac{da}{dt} \quad \frac{-30}{10\sqrt{3}} = \frac{da}{dt} \quad \frac{-3}{\sqrt{3}} = \frac{da}{dt} = -\sqrt{3}$

5.



$\frac{da}{dt} = 10$

$10 \times 5 = 50$

$\frac{db}{dt} = 30 \quad 30 \times 2 = 60$

$c^2 = a^2 + b^2$

$c^2 = 50^2 + 60^2$

$c^2 = 2500 + 3600$

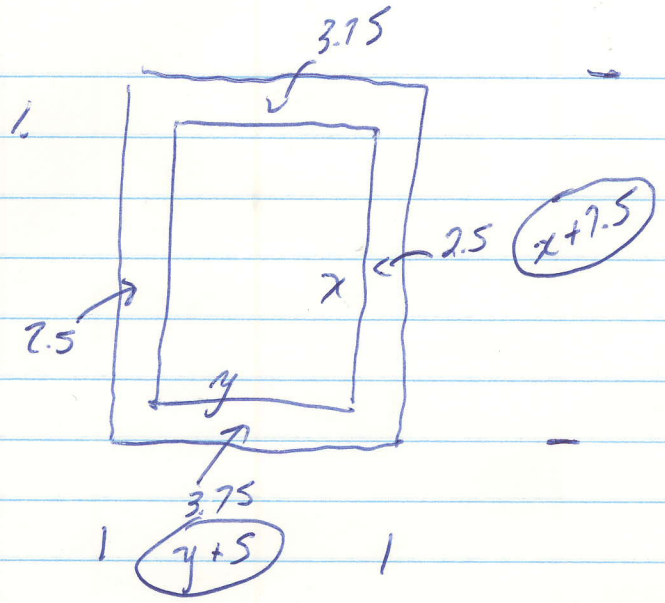
$c^2 = 6100 \quad c = \sqrt{6100}$

$2c\frac{dc}{dt} = 2a\frac{da}{dt} + 2b\frac{db}{dt} \qquad c\frac{dc}{dt} = a\frac{da}{dt} + b\frac{db}{dt}$

$\sqrt{6100}\frac{dc}{dt} = 50(10) + 60(30) \qquad \sqrt{6100}\frac{dc}{dt} = 500 + 1800$

$\sqrt{6100}\frac{dc}{dt} = 2300 \qquad \frac{dc}{dt} = \frac{2300}{\sqrt{6100}} = \frac{23}{61}\sqrt{6100}$

1.



$A = xy$ $\quad A = (x+7.5)(y+5)$

$150 = xy$ $\quad A = xy + 5x + 7.5y + 37.5$

$\frac{150}{x} = y$ $\quad A = x\left(\frac{150}{x}\right) + 5x + 7.5\left(\frac{150}{x}\right) + 37.5$

$A = 150 + 5x + \frac{1125}{x} + 37.5$

$A = 187.5 + 5x + 1125x^{-1}$

$A' = 5 - 1125x^{-2}$

$0 = 5 - 1125x^{-2}$ $\quad 0 = 5 - \frac{1125}{x^2}$

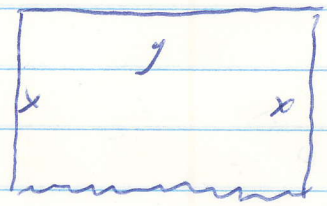
Domain - difficult to determine

2nd derivative $0 = 1125 \cdot 2x^{-3}$ $\quad 0 = 5x^2 - 1125$ $\quad 5x^2 = 1125$

positive $\therefore$ a max. $\quad x^2 = 225$ $\quad x = 15$

$\therefore y = 10$ dimensions (22.5×15)

2.



domain $[0, 1000]$

$2x + y = 2000$ $\quad y = 2000 - 2x$

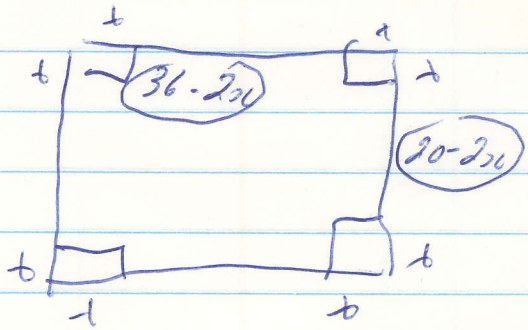
$A = xy$ $\quad A = x(2000 - 2x)$

$A = 2000x - 2x^2$ $\quad A = 2000(0) - 2(0)^2 = 0$

$x = 500$ $\quad A' = 2000 - 4x$ $\quad A = 2000(500) - 2(500)^2 = 500{,}000$

$y = 1000$ $\quad x = 500$ $\quad A = 2000(1000) - 2(1000)^2 = 0$

3.



$V = lwh$ $\quad V = (36-2x)(20-x)(x)$

$V = 4x^3 - 112x^2 + 720x$

$V' = 12x^2 - 224x + 720$

$V' = 4(3x^2 - 56x + 180)$

$x = \frac{56 \pm \sqrt{1264}}{6}$ $\quad x = 4.12$ $\quad x = 14.54$

Domain $[0, 10]$

eliminate $x = 14.54$

$V = 4(0)^3 - 112(0)^2 + 720(0) = 0$

$V = 4(4.12)^3 - 112(4.12)^2 + 720(4.12) = 1345$

$V = 4(10)^3 - 112(10)^2 + 720(10) = 0$

#5

4. 

Surface area: 2 circles: $\pi r^2 \times 2 = 2\pi r^2$

rectangle: circumference × height

$2\pi r \times h = 2\pi rh$

$SA = 2\pi r^2 + 2\pi rh$

$SA = 2\pi r^2 + 2\pi r\left(\frac{350}{\pi r^2}\right)$

$SA = 2\pi r^2 + 700r^{-1}$

$SA' = 4\pi r - 700r^{-2}$

$SA' = 4\pi r - \frac{700}{r^2}$

$SA' = \frac{4\pi r^3 - 700}{r^2}$

$V = \pi r^2 h$

$350 = \pi r^2 h$

$\frac{350}{\pi r^2} = h$

$0 = \frac{4\pi r^3 - 700}{r^2}$

$4\pi r^3 - 700 = 0$

$r^3 = \frac{700}{4\pi}$

$r = \sqrt[3]{\frac{700}{4\pi}} = 3.819$

height $= \frac{350}{\pi(3.819)^2} = 7.638$