Basic Integration and Substitution

$$\int (x-2)^{\frac{3}{2}}\,dx = \int u^{\frac{3}{2}}\,du = \frac{u^{\frac{5}{2}}}{\frac{5}{2}} = \frac{2}{5}u^{\frac{5}{2}} = \frac{2}{5}(x-2)^{\frac{5}{2}} + c$$

1. $u = (x-2) \Rightarrow du = dx$

2. $$\int \frac{dx}{\sqrt{x+3}} = \int (x+3)^{-\frac{1}{2}}\,dx = \int u^{-\frac{1}{2}}\,du = \frac{u^{\frac{1}{2}}}{\frac{1}{2}} = 2u^{\frac{1}{2}} = 2(x+3)^{\frac{1}{2}} + c$$

$u = (x+3) \Rightarrow du = dx$

3. $$\int \sqrt{2-3x}\,dx = \int (2-3x)^{\frac{1}{2}}\,dx = -\frac{1}{3}\int u^{\frac{1}{2}}\,du = -\frac{1}{3}\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}} = -\frac{2}{9}u^{\frac{3}{2}} = -\frac{2}{9}(2-3x)^{\frac{3}{2}} + c$$

$u = (2-3x) \Rightarrow du = -3dx \Rightarrow -\frac{1}{3}du = dx$

4. $$\int (2x^2+3)^{\frac{1}{3}} x\,dx = \frac{1}{4}\int u^{\frac{1}{3}}\,du = \frac{1}{4}\cdot\frac{u^{\frac{4}{3}}}{\frac{4}{3}} = \frac{3}{16}u^{\frac{4}{3}} = \frac{3}{16}(2x^2+3)^{\frac{4}{3}} + c$$

$u = (2x^2+3)dx \Rightarrow du = 4xdx \Rightarrow \frac{1}{4}du = xdx$

5. $$\int \sqrt{1+y^4}\,y^3\,dy = \int (1+y^4)^{\frac{1}{2}}y^3\,dy = \frac{1}{4}\int u^{\frac{1}{2}}\,dy = \frac{1}{4}\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}} = \frac{1}{6}u^{\frac{3}{2}} = \frac{1}{6}(1+y^4)^{\frac{3}{2}} + c$$

$u = (1+y^4) \Rightarrow du = 4y^3\,dy \Rightarrow \frac{1}{4}du = y^3\,dy$

6. $$\int (4-x^2)^2 x^2\,dx = \int (16-8x^2+x^4)x^2\,dx = \int (16x^2-8x^4+x^6)\,dx = 16\int x^2\,dx - 8\int x^4\,dx + \int x^6\,dx =$$
$$16\cdot\frac{x^3}{3} - 8\cdot\frac{x^5}{5} + \frac{x^7}{7} = \frac{16}{3}x^3 - \frac{8}{5}x^5 + \frac{x^7}{7} + c$$

7.

$$\int \frac{xdx}{\left(x^2+4\right)^3} = \int \left(x^2+4\right)^{-3} xdx = \frac{1}{2}\int u^{-3}du = \frac{1}{2}\cdot\frac{u^{-2}}{-2} = -\frac{1}{4}u^{-2} = -\frac{1}{4}\left(x^2+4\right)^{-2}+c$$

$$u=\left(x^2+4\right) \Rightarrow du = 2xdx \Rightarrow \frac{1}{2}du = xdx$$

8.

$$\int \left(x^2-x\right)(2x-1)dx = \int udu = \frac{u^2}{2} = \frac{\left(x^2-x\right)^2}{2}+c$$

$$u=\left(x^2-x\right) \Rightarrow du = (2x-1)dx$$

9.

$$\int \frac{3tdt}{\sqrt[3]{t^2+3}} = 3\int \left(t^2+3\right)^{-\frac{1}{3}}tdt = 3\cdot\frac{1}{2}\int u^{-\frac{1}{3}}du = \frac{3}{2}\cdot\frac{u^{\frac{2}{3}}}{\frac{2}{3}} = \frac{9}{4}u^{\frac{2}{3}} = \frac{9}{4}\left(t^2+3\right)^{\frac{2}{3}}+c$$

$$u=\left(t^2+3\right) \Rightarrow du = 2tdt \Rightarrow \frac{1}{2}du = tdt$$

10.

$$\int \frac{(x+1)dx}{\sqrt{x^2+2x-4}} = \int \left(x^2+2x-4\right)^{-\frac{1}{2}}(x+1)dx = \frac{1}{2}\int u^{-\frac{1}{2}}du = \frac{1}{2}\cdot\frac{u^{\frac{1}{2}}}{\frac{1}{2}} = u^{\frac{1}{2}} = \left(x^2+2x-4\right)^{\frac{1}{2}}+c$$

$$u=\left(x^2+2x-4\right) \Rightarrow du = (2x+2)dx \Rightarrow du = 2(x+1)dx \Rightarrow \frac{1}{2}du = (x+1)dx$$

11.

$$\int \sqrt{x}\,(3-5x)dx = \int x^{\frac{1}{2}}(3-5x)dx = \int \left(3x^{\frac{1}{2}}-5x^{\frac{3}{2}}\right)dx = 3\int x^{\frac{1}{2}}dx - 5\int x^{\frac{3}{2}}dx =$$

$$3\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 5\cdot\frac{x^{\frac{5}{2}}}{\frac{5}{2}} = 2x^{\frac{3}{2}} - 2x^{\frac{5}{2}}+c$$

12.

$$\int \frac{(x+1)(x-2)}{\sqrt{x}}dx = \int \frac{x^2-x-2}{x^{\frac{1}{2}}}dx = \int \frac{x^2}{x^{\frac{1}{2}}} - \frac{x}{x^{\frac{1}{2}}} - \frac{2}{x^{\frac{1}{2}}}dx = \int x^{\frac{3}{2}} - x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}dx =$$

$$\int x^{\frac{3}{2}}dx - \int x^{\frac{1}{2}}dx - 2\int x^{-\frac{1}{2}}dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 2\cdot\frac{x^{\frac{1}{2}}}{\frac{1}{2}} = \frac{2}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} - 4x^{\frac{1}{2}}+c$$

13.
$$\int \frac{dx}{3x+1} = \int (3x+1)^{-1} dx = \frac{1}{3}\int u^{-1} du = \frac{1}{3}\ln u = \frac{1}{3}\ln(3x+1)$$

$$u = (3x+1) \Rightarrow du = 3dx \Rightarrow \frac{1}{3}du = dx$$

14.
$$\int \frac{3xdx}{x^2+2} = 3\int (x^2+2)^{-1} xdx = 3\cdot\frac{1}{2}\int u^{-1} du = \frac{3}{2}\ln u = \frac{3}{2}\ln(x^2+2)+c$$

$$u = (x^2+2) \Rightarrow du = 2xdx \Rightarrow \frac{1}{2}du = dx$$

15.
$$\int \frac{x^2 dx}{1-x^3} = \int (1-x^3)^{-1} x^2 dx = -\frac{1}{3}\int u^{-1} du = -\frac{1}{3}\ln u = -\frac{1}{3}\ln(1-x^3)+c$$

$$u = (1-x^3) \Rightarrow du = -3x^2 dx \Rightarrow -\frac{1}{3}du = x^2 dx$$

$$\int \frac{x-1}{x+1} dx = \int 1 - \frac{2}{x+1} dx = \int dx - 2\int \frac{1}{(x+1)} dx = \int dx - 2\int (x+1)^{-1} dx =$$

16. $\int dx - 2\int (u)^{-1} dx = x - 2\ln u = x - 2\ln(x+1)+c$

$$u = (x+1) \Rightarrow du = dx$$

$$\int \frac{x^2+2x+2}{x+2} dx = \int x + \frac{2}{x+2} dx = \int xdx + 2\int \frac{1}{x+2} dx = \int xdx + 2\int (x+2)^{-1} dx =$$

17. $\int xdx + 2\int u^{-1} du = \frac{x^2}{2} + 2\ln u = \frac{x^2}{2} + 2\ln(x+2)+c$

$$u = (x+2) \Rightarrow du = dx$$

18.
$$\int \frac{x+1}{x^2+2x+2} dx = \int (x+1)(x^2+2x+2)^{-1} dx = \frac{1}{2}\int u^{-1} du = \frac{1}{2}\ln u = \frac{1}{2}\ln(x^2+2x+2)+c$$

$$u = (x^2+2x+2) \Rightarrow du = (2x+2)dx \Rightarrow du = 2(x+1)dx \Rightarrow \frac{1}{2}du = (x+1)dx$$

19.
$$\int e^{4x} dx = \frac{1}{4}\int e^u du = \frac{1}{4}e^u = \frac{1}{4}e^{4x}+c$$

$$u = 4x \Rightarrow du = 4dx \Rightarrow \frac{1}{4}du = dx$$

20. $\int \dfrac{e^{\frac{1}{x^2}}}{x^3}dx = \int e^{x^{-2}} x^{-3} dx = -\dfrac{1}{2}\int e^u du = -\dfrac{1}{2}e^u = -\dfrac{1}{2}e^{x^{-2}}+c$

$u = x^{-2} \Rightarrow du = -2x^{-3}dx \Rightarrow -\dfrac{1}{2}du = dx$

21. $\int e^{-x^2+2}xdx = -\dfrac{1}{2}\int e^u du = -\dfrac{1}{2}\cdot e^u = -\dfrac{1}{2}\cdot e^{-x^2+2}+c$

$u = -x^2+2 \Rightarrow du = -2xdx \Rightarrow -\dfrac{1}{2}du = xdx$

22. $\int \dfrac{e^{2x}}{e^{2x}+3}dx = \int e^{2x}\left(e^{2x}+3\right)^{-1} dx = \dfrac{1}{2}\int u^{-1}du = \dfrac{1}{2}\ln u = \dfrac{1}{2}\ln\left(e^{2x}+3\right)+c$

$u = \left(e^{2x}+3\right) \Rightarrow du = 2e^{2x}dx \Rightarrow \dfrac{1}{2}du = e^{2x}dx$

23. $\int \sin(2x)dx = \dfrac{1}{2}\int \sin u\, du = \dfrac{1}{2}\cdot -\cos u = -\dfrac{1}{2}\cos(2x)$

$u = 2x \Rightarrow du = 2dx \Rightarrow \dfrac{1}{2}du = dx$

24. $\int \sin^3 x \cos x dx = \int (\sin x)^3 \cos dx = \int u^3 du = \dfrac{u^4}{4} = \dfrac{(\sin x)^4}{4}+c$

$u = \sin x \Rightarrow du = \cos dx$

25. $\int \cos^4 x \sin x dx = \int (\cos x)^4 \sin x dx = -\int u^4 du = -\dfrac{u^5}{5} = -\dfrac{(\cos x)^5}{5}+c$

$u = \cos x \Rightarrow du = -\sin xdx \Rightarrow -du = \sin xdx$

26. $\int \dfrac{\sec^5 x}{\csc x}dx = \int \dfrac{\sin x}{(\cos x)^5}dx = \int \sin x(\cos x)^{-5}dx = -\int u^{-5} du = -\dfrac{u^{-4}}{-4} = \dfrac{(\cos x)^{-4}}{4}+c$

$u = \cos x \Rightarrow du = -\sin xdx \Rightarrow -du = \sin dx$