DERIVATIVES - TRIG FUNCTIONS

1. $f(x) = 4(\cos x)$

   $f'(x) = -4\sin x$

2. $f(x) = 3x(\sin x)$

   $f'(x) = 3\sin x + (\cos x)3x$

   $f'(x) = 3(\sin x + x\cos x)$

3. $f(x) = x^3(\sin x)$

   $f'(x) = 3x^2(\sin x) + (\cos x)x^3$

   $f''(x) = x^2(3\sin x + x\cos x)$

4. $f(x) = x - x^2(\cos x)$

   $f'(x) = 1 - \left[2x\cos x + (-\sin x)x^2\right]$

   $f'(x) = 1 - x[2\cos x - x\sin x]$

5. $f(x) = \dfrac{\sin\ x}{x}$

   $f'(x) = \dfrac{\cos x(x) - \sin x}{x^2}$

   $f'(x) = \dfrac{x\cos x - \sin x}{x^2}$

6. $f(x) = \dfrac{x^2}{\cos x}$

   $f'(x) = \dfrac{2x\cos x - (-\sin x)x^2}{(\cos x)^2}$

   $f'(x) = \dfrac{x(2\cos x + x\sin x)}{(\cos x)^2}$

7. $f(x) = 2x(\cos x) + x^2(\sin x)$

   $f'(x) = \left(2\cos x + (-\sin x)(2x)\right) + \left(2x(\sin x) + \cos x(x^2)\right)$

   $f'(x) = 2\cos x - (\sin x)(2x) + 2x(\sin x) + \cos x(x^2)$

   $f'(x) = \cos x(2 + x^2)$

8. $f(x) = \sin x \cot x$

   $f'(x) = \sin x \cdot \dfrac{\cos x}{\sin x}$

   $f'(x) = \cos x$

   $f'(x) = -\sin x$

9. $f(x) = 3x^2(\sin x) - x^3(\cos x)$

   $f'(x) = \left[6x(\sin x) + (\cos x)(3x^2)\right] - \left[3x^2(\cos x) + (-\sin x)(x^3)\right]$

   $f'(x) = 6x(\sin x) + (\cos x)(3x^2) - 3x^2(\cos x) + (\sin x)(x^3)$

   $f'(x) = x\sin x(6 + x^2)$

10. $f(x) = (\sin x - \cos x)^2$

    $f'(x) = 2(\sin x - \cos x) \cdot (\cos x - (-\sin x))$

    $f'(x) = 2(\sin x - \cos x) \cdot (\cos x + \sin x)$

    $f'(x) = 2(\sin^2 x - \cos^2 x)$ or $2(\sin^2 x - (1 - \sin^2 x) = 2(2\sin^2 x - 1))$ or

    $2((1 - \cos^2 x) - \cos^2 x) = 2(1 - 2\cos^2 x)$

11. $f(x)=\dfrac{(1-\cos x)}{(1+\cos x)}$

$$f'(x)=\frac{-(-\sin x)(1+\cos x)-(-\sin x)(1-\cos x)}{(1+\cos x)^2}$$

$$f'(x)=\frac{\sin x+\sin x\cos x+\sin x-\sin x\cos x}{(1+\cos x)^2}$$

$$f'(x)=\frac{2\sin x}{(1+\cos x)^2}$$

12. $f(x)=\dfrac{\cos x}{(1-\sin x)}$

$$f'(x)=\frac{-\sin x-(-\cos x)\cos x}{(1-\sin x)^2}$$

$$f'(x)=\frac{-\sin x+\cos^2 x}{(1-\sin x)^2}$$

$$f'(x)=\frac{-\sin x+1-\sin^2 x}{(1-\sin x)^2}$$

$$f'(x)=\frac{-\sin^2 x-\sin x+1}{(1-\sin x)^2}$$

13. $f(x)=\dfrac{(x+\sin x)}{(1-\cos x)}$

$$f'(x)=\frac{(1+\cos x)(1-\cos x)-(-(-\sin x))(x+\sin x)}{(1-\cos x)^2}$$

$$f'(x)=\frac{1-\cos^2 x-x\sin x-\sin^2 x}{(1-\cos x)^2}$$

$$f'(x)=\frac{1-(1-\sin^2 x)-x\sin x-\sin^2 x}{(1-\cos x)^2}$$

$$f'(x)=\frac{1-1+\sin^2 x-x\sin x-\sin^2 x}{(1-\cos x)^2}$$

$$f'(x)=\frac{-x\sin x}{(1-\cos x)^2}$$

14. $f(x)=(\sin^2 x+\cos^2 x)^3$

$$f'(x)=3(\sin^2 x+\cos^2 x)^2\left(2\sin x\cos x+2\cos x(-\sin x)\right)$$

$$f'(x)=3(\sin^2 x+\cos^2 x)^2\left(2\sin x\cos x-2\cos x\sin x\right)$$

$$f'(x)=3(\sin^2 x+\cos^2 x)^2(0)$$

$$f'(x)=0$$

15. $f(x)=\sin(5x+2)$

$$f'(x)=\cos(5x+2)\cdot 5=5\cos(5x+2)$$

16. $f(x)=\cos(4-3x)$

$$f'(x)=-\sin(4-3x)\cdot(-3)=3\sin x(4-3x)$$

17. $f(x)=\cos(2x^2-3x+1)$

$$f'(x)=\sin(2x^2-3x+1)\cdot(4x-3)$$

18. $f(x)=\sin(2x^5)$

$$f'(x)=-\cos(2x^5)\cdot(10x^4)=-(10x^4)\cos(2x^5)$$

19. $f(x)=\cos^4(3x)$

$$f'(x)=4\cos^3(3x)\cdot(-\sin(3x))\cdot 3$$

$$f'(x)=-12\sin(3x)\cos^3(3x)\Rightarrow$$

$$f'(x)=-6\sin(6x)\cos^2(3x)$$

20. $f(x)=\sin^3(x^4)$

$$f'(x)=3\sin^2(x^4)\cdot\cos(x^4)\cdot 4x^3$$

$$f'(x)=12x^3\sin^2(x^4)\cdot\cos(x^4)\Rightarrow$$

$$f'(x)=6x^3\sin(x^4)\cdot\sin(2x^4)$$

21. $f(x)=\sin^2 x \ + \ \sin x^2$

$f'(x)=2\sin x\cos x+\cos x^2\cdot 2x$

$f'(x)=\sin(2x)+2x\cos x^2$

22. $f(x)=(x^4+\cos^4 x)^4$

$f'(x)=4(x^4+\cos^4 x)^3\cdot(4x^3+4\cos^3 x(-\sin x))$

$f'(x)=4(x^4+\cos^4 x)^3\cdot 4(x^3-\cos^3 x\sin x)$

$f'(x)=16(x^4+\cos^4 x)^3(x^3-\cos^3 x\sin x)$

23. $f(x)=\dfrac{\sin\ 3x}{\sin\ 3x+\cos\ 3x}$

$$f'(x)=\frac{\cos(3x)\cdot 3\left(\sin(3x)+\cos(3x)\right)-\left(\cos(3x)\cdot 3+(-\sin(3x)\cdot 3\right)\sin(3x)}{\left(\sin(3x)+\cos(3x)\right)^2}$$

$$f'(x)=\frac{3\cos(3x)\sin(3x)+3\cos^2(3x)-3\sin(3x)\cos(3x)+3\sin^2(3x)}{\left(\sin(3x)+\cos(3x)\right)^2}$$

$$f'(x)=\frac{3\sin^2(3x)+3\cos^2(3x)}{\left(\sin(3x)+\cos(3x)\right)^2}$$

$$f'(x)=\frac{3(\sin^2(3x)+\cos^2(3x))}{\sin^2(3x)+2\sin(3x)\cos(3x)+\cos^2(3x)}=\frac{3(1)}{1+2\sin(3x)\cos(3x)}=\frac{3}{1+\sin 2(3x)}$$

24. $f(x)=\dfrac{\sin(3x+4)}{(3x+4)}$

$$f'(x)=\frac{\cos(3x+4)\cdot 3\cdot(3x+4)-3\cdot\sin(3x+4)}{(3x+4)^2}$$

$$f'(x)=\frac{3\left[(3x+4)\cos(3x+4)-\sin(3x+4)\right]}{(3x+4)^2}$$

25. $f(x)=(\sin^3 x)(\cos^4 x)$

$f'(x)=3\sin^2 x\cdot\cos x\cdot(\cos^4 x)+4\cos^3 x\cdot(-\sin x)\cdot(\sin^3 x)$

$f'(x)=3\sin^2 x\cdot\cos^5 x-4\cos^3 x\sin^4 x$

$f'(x)=\sin^2 x\cos^3 x(3\cos^2 x-4\sin^2 x)$

26. $f(x)=\sin(\cos^5 x)$

$f'(x)=\cos\left[(\cos^5 x)\right]\cdot 5(\cos^4 x)\cdot 4\cos^3 x\cdot(-\sin x)$

27. $f(x)=(\cos x^{1/3}-\sin x^{1/3})^3$

$$f'(x)=3(\cos x^{1/3}-\sin x^{1/3})^2\left(-\sin x^{1/3}\cdot \tfrac{1}{3}x^{-2/3}-\cos x^{1/3}\cdot \tfrac{1}{3}x^{-2/3}\right)$$
$$f'(x)=3(\cos x^{1/3}-\sin x^{1/3})^2\cdot -\tfrac{1}{3}x^{-2/3}\cdot\left(\sin x^{1/3}+\cos x^{1/3}\right)$$
$$f'(x)=-x^{-2/3}(\cos x^{1/3}-\sin x^{1/3})^2\left(\sin x^{1/3}+\cos x^{1/3}\right)$$

28. $f(x)=\ln(\cos x)$

$$f'(x)=\frac{1}{\cos x}\cdot -\sin x=-\tan x$$

29. $f(x)=\cos(\ln x)$

$$f'(x)=-\sin(\ln x)\cdot\frac{1}{x}=-\frac{1}{x}\sin(\ln x)$$

30. $f(x)=\ln(x^3\sin x)$

$$f'(x)=\frac{1}{(x^3\sin x)}\cdot\left[3x^2\sin x+\cos x\cdot x^3\right]$$
$$f'(x)=\frac{1}{(x^3\sin x)}\cdot x^2\left[3\sin x+x\cos x\right]$$
$$f'(x)=\frac{1}{(x\sin x)}\left[3\sin x+x\cos x\right]$$

31. $f(x)=(\ln\sin x)^3$

$$f'(x)=3(\ln\sin x)^2\cdot\frac{1}{\sin x}\cdot\cos x$$
$$f'(x)=3(\ln\sin x)^2\cdot\cot x$$

32. $f(x)=x\sin x$

$$f'(x)=\sin x+\cos x\cdot x$$

33. $f(x)=(\sin x)\cos x$

$$f'(x)=\cos x\cdot\cos x+-\sin x\cdot\sin x$$
$$f'(x)=\cos^2 x-\sin^2 x=\cos(2x)$$

34. $f(x)=\tan(8x+3)=\dfrac{\sin(8x+3)}{\cos(8x+3)}$

$$f'(x)=\frac{\cos(8x+3)\cdot 8\cdot\cos(8x+3)-(-\sin(8x+3))\cdot 8\cdot\sin(8x+3)}{\left(\cos(8x+3)\right)^2}$$
$$f'(x)=\frac{8\cos^2(8x+3)+8\sin^2(8x+3)}{\left(\cos(8x+3)\right)^2}$$
$$f'(x)=\frac{8\left(\cos^2(8x+3)+\sin^2(8x+3)\right)}{\left(\cos(8x+3)\right)^2}$$
$$f'(x)=\frac{8}{\left(\cos(8x+3)\right)^2}$$

But if we were to take the derivative of “ tan x” we would get

$$f(x)=\tan x=\frac{\sin x}{\cos x}$$

$$f'(x)=\frac{\cos x\cos x-(-\sin x)\sin x}{(\cos x)^2}=\frac{\cos^2 x+\sin^2 x}{\cos^2 x}=\frac{1}{\cos^2 x}=\sec^2 x$$

$34.\ f(x)=\tan(8x+3)$

$f'(x)=\sec^2(8x+3)\cdot 8=8\sec^2(8x+3)$

$$f(x)=\csc x=\frac{1}{\sin x}=(\sin x)^{-1}$$

$$f'(x)=-1(\sin x)^{-2}\cos x=-\frac{\cos x}{\sin^2 x}=-\cot x\csc x$$

$$f(x)=\sec x=\frac{1}{\cos x}=(\cos x)^{-1}$$

$$f'(x)=-1(\cos x)^{-2}(-\sin x)=\frac{\sin x}{\cos^2 x}=\tan x\sec x$$

$$35.\ f(x)=\csc(x^2+4)=\frac{1}{\sin(x^2+4)}=\sin^{-1}(x^2+4)$$

$$f'(x)=-1\sin^{-2}(x^2+4)\cos(x^2+4)\cdot 2x$$

$$f'(x)=-\frac{2x\cos(x^2+4)}{\sin^2(x^2+4)}$$

$$36.\ f(x)=\tan^2 x\sec^3 x=\frac{\sin^2 x}{\cos^2 x}\cdot\frac{1}{\cos^3 x}=\frac{\sin^2 x}{\cos^5 x}$$

$$f'(x)=\frac{2\sin x\cos x\cdot\cos^5 x-5\cos^4 x\sin x\cdot\sin^2 x}{(\cos^5 x)^2}$$

$$f'(x)=\frac{2\sin x\cos^6 x-5\cos^4 x\sin^3 x}{\cos^{10} x}$$

$$f'(x)=\frac{\sin x\cos^4 x(2\cos^2 x-5\sin^2 x)}{\cos^{10} x}$$

$$f'(x)=\frac{\sin x(2\cos^2 x-5\sin^2 x)}{\cos^6 x}$$

$35.\ f(x)=\csc(x^2+4)$

$f'(x)=-\cot(x^2+4)\csc(x^2+4)\cdot 2x$

$36.\ f(x)=\tan^2 x\sec^3 x$

$f'(x)=2\tan x\sec^2 x\cdot\sec^3 x+3\sec^2 x\cdot\tan x\sec x\cdot\tan^2 x$

$f'(x)=2\tan x\sec^5 x+3\sec^3 x\tan^3 x$

$f'(x)=\tan x\sec^3 x(2\sec^2 x+3\tan^2 x)$

$$37.\ f(x)=e^{-3x}\ \ \tan x^{(1/2)}=\frac{e^{-3x}\cdot\sin x^{1/2}}{\cos x^{1/2}}$$

$$f'(x)=\frac{\left(e^{-3x}\cdot(-3)\cdot\sin x^{1/2}+\cos x^{1/2}\cdot \tfrac{1}{2}x^{-1/2}\cdot e^{-3x}\right)\cos x^{1/2}-(-\sin x^{1/2})\cdot \tfrac{1}{2}x^{-1/2}\cdot e^{-3x}\cdot\sin x^{1/2}}{\left(\cos x^{1/2}\right)^2}$$

$$f'(x)=\frac{-3e^{-3x}\sin x^{1/2}\cos x^{1/2}+\tfrac{1}{2}x^{-1/2}e^{-3x}\cos^2 x^{1/2}+\tfrac{1}{2}x^{-1/2}\cdot e^{-3x}\sin^2 x^{1/2}}{\cos^2 x^{1/2}}$$

$$f'(x)=\frac{-3e^{-3x}\sin x^{1/2}\cos x^{1/2}+\tfrac{1}{2}x^{-1/2}e^{-3x}\left(\cos^2 x^{1/2}+\sin^2 x^{1/2}\right)}{\cos^2 x^{1/2}}$$

$$f'(x)=\frac{e^{-3x}\left(-3\sin x^{1/2}\cos x^{1/2}+\tfrac{1}{2}x^{-1/2}\right)}{\cos^2 x^{1/2}}$$

$$37.\ f(x)=e^{-3x}\ \ \tan x^{(1/2)}$$

$$f'(x)=e^{-3x}\cdot -3\cdot\tan x^{(1/2)}+\sec^2 x^{(1/2)}\cdot \tfrac{1}{2}x^{-1/2}\cdot e^{-3x}$$

$$f'(x)=e^{-3x}\left(-3\cdot\tan x^{(1/2)}+\tfrac{1}{2}x^{-1/2}\sec^2 x^{(1/2)}\right)$$

$$38.\ f(x)=\frac{(\sec^2 x)}{\tan\ (2x+1)}=\frac{\frac{1}{\cos^2 x}}{\frac{\sin(2x+1)}{\cos(2x+1)}}\ \ =\frac{\cos(2x+1)}{\cos^2 x\left(\sin(2x+1)\right)}$$

$$f'(x)=\frac{-\sin(2x+1)\cdot 2\cdot\cos^2 x\left(\sin(2x+1)\right)-\left[2\cos x\cdot\left(-\sin x\right)\cdot\left(\sin(2x+1)\right)+\cos(2x+1)\cdot 2\cdot\cos^2 x\right]\cos(2x+1)}{\left[\cos^2 x\left(\sin(2x+1)\right)\right]^2}$$

$$f'(x)=\frac{-2\sin^2(2x+1)\cos^2 x+2\sin x\cos x\sin(2x+1)\cos(2x+1)-2\cos^2 x\cos^2(2x+1)}{\cos^4 x\left(\sin^2(2x+1)\right)}$$

$$f'(x)=\frac{-2\cos^2 x\left[\sin^2(2x+1)+\cos^2(2x+1)\right]+2\sin x\cos x\sin(2x+1)\cos(2x+1)}{\cos^4 x\left(\sin^2(2x+1)\right)}$$

$$f'(x)=\frac{-2\cos^2 x+2\sin x\cos x\sin(2x+1)\cos(2x+1)}{\cos^4 x\left(\sin^2(2x+1)\right)}$$

$$38.\ f(x)=\frac{(\sec^2 x)}{\tan\ (2x+1)}$$

$$f'(x)=\frac{2\sec x\cdot\tan x\sec x\cdot\tan(2x+1)-\sec^2(2x+1)\cdot 2\cdot\sec^2 x}{(\tan(2x+1))^2}$$

$$f'(x)=\frac{2\sec^2 x\left(\tan x\tan(2x+1)-\sec^2(2x+1)\right)}{(\tan(2x+1))^2}$$

$$39.\ f(x)=\tan^3 6x$$

$$f'(x)=3\tan^2 6x\cdot\sec^2 6x\cdot 6$$

$$f'(x)=18\tan^2 6x\cdot\sec^2 6x$$

$$40.\ f(x)=\ln\ln\sec^2 x$$

$$f'(x)=\frac{1}{\ln\sec^2 x}\cdot\frac{1}{\sec^2 x}\cdot 2\sec x\cdot\tan x\sec x$$

$$f'(x)=\frac{1}{\ln\sec^2 x}\cdot\frac{1}{\sec^2 x}\cdot 2\sec^2 x\cdot\tan x$$

$$39.\ f(x)=\frac{\sin^3 6x}{\cos^3 6x}$$

$$f'(x)=\frac{3\sin^2 6x\cdot\cos 6x\cdot 6\cdot\cos^3 6x-3\cos^2 6x\cdot(-\sin 6x)\cdot 6\cdot\sin^3 6x}{\left(\cos^3 6x\right)^2}$$

$$f'(x)=\frac{18\sin^2 6x\cos^2 6x\left(\cos^2 6x+\sin^2 6x\right)}{\cos^6 6x}$$

$$f'(x)=\frac{18\sin^2 6x}{\cos^4 6x}$$

$$40.\ f(x)=\ln\ln\left(\frac{1}{\cos^2 x}\right)$$

$$f'(x)=\frac{1}{\ln\left(\frac{1}{\cos^2 x}\right)}\cdot\frac{1}{\left(\frac{1}{\cos^2 x}\right)}\cdot -2\cos^{-3} x\cdot -\sin x$$

$$f'(x)=\frac{1}{\ln\left(\frac{1}{\cos^2 x}\right)}\cdot\frac{1}{\left(\frac{1}{\cos^2 x}\right)}\cdot\frac{2\sin x}{\cos^2 x}$$