Related Rates

1. If $y=1$ then $x^2+3x(1)+(1)^2=1 \Rightarrow x^2+3x=0 \Rightarrow x(x+3)=0 \Rightarrow x=0, x=-3$

$$2x\frac{dx}{dt}+\left(3y\frac{dx}{dt}+3x\frac{dy}{dt}\right)+2y\frac{dy}{dt}=0$$

$$\text{For } x=0, y=1, \frac{dy}{dt}=2 \Rightarrow 2(0)\frac{dx}{dt}+\left(3(1)\frac{dx}{dt}+3(0)(2)\right)+2(1)(2)=0 \Rightarrow 3\frac{dx}{dt}=-4 \Rightarrow \frac{dx}{dt}=-\frac{4}{3}$$

$$\text{For } x=-3, y=1, \frac{dy}{dt}=2 \Rightarrow 2(-3)\frac{dx}{dt}+\left(3(1)\frac{dx}{dt}+3(-3)(2)\right)+2(1)(2)=0 \Rightarrow -3\frac{dx}{dt}=14 \Rightarrow \frac{dx}{dt}=-\frac{14}{3}$$

2. $V=\frac{4}{3}\pi r^3$ and $r=\frac{d}{2} \Rightarrow V=\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \Rightarrow V=\frac{4}{3}\pi\frac{d^3}{8} \Rightarrow V=\frac{\pi}{6}d^3$

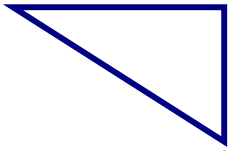
$$\frac{dV}{dt}=\frac{\pi}{6}\cdot 3d^2\frac{dd}{dt} \Rightarrow \frac{dV}{dt}=\frac{\pi}{2}d^2\frac{dd}{dt}$$

$$\text{For } d=10cm, \frac{dV}{dt}=1cm^3/\min \Rightarrow 1cm^3/\min=\frac{\pi}{2}(10cm)^2\frac{dd}{dt} \Rightarrow \frac{1}{50\pi}cm/\min=\frac{dd}{dt}$$

3. $S=4\pi r^2$ and $r=\frac{d}{2} \Rightarrow S=4\pi\left(\frac{d}{2}\right)^2 \Rightarrow S=\pi d^2 \Rightarrow \frac{dS}{dt}=2\pi d\frac{dd}{dt}$

$$Ford=10cm, \frac{dS}{dt}=1cm^2/\min \Rightarrow 1cm^2/\min=2\pi\cdot 10cm\frac{dd}{dt} \Rightarrow \frac{1}{20\pi}cm/\min=\frac{dd}{dt}$$

4.



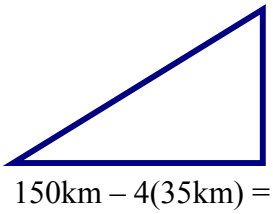
$$c^2=a^2+b^2$$
$$c^2=(120mi)^2+(50mi)^2 \Rightarrow c=130mi$$

4. $c^2=a^2+b^2 \Rightarrow 2c\frac{dc}{dt}=2a\frac{da}{dt}+2b\frac{db}{dt} \Rightarrow c\frac{dc}{dt}=a\frac{da}{dt}+b\frac{db}{dt}$

$$\text{For } a=120mi, b=50mi, c=130mi, \frac{da}{dt}=60mi/hr, \frac{db}{dt}=25mi/hr, \frac{dc}{dt}=?$$

$$130mi\cdot\frac{dc}{dt}=120mi\cdot 60mi/hr+50mi\cdot 25mi/hr \Rightarrow 130mi\cdot\frac{dc}{dt}=8450mi^2/hr \Rightarrow \frac{dc}{dt}=65mi/hr$$

5.



$$c^2=a^2+b^2$$
$$c^2=(10km)^2+(100km)^2 \Rightarrow c=100.5km$$

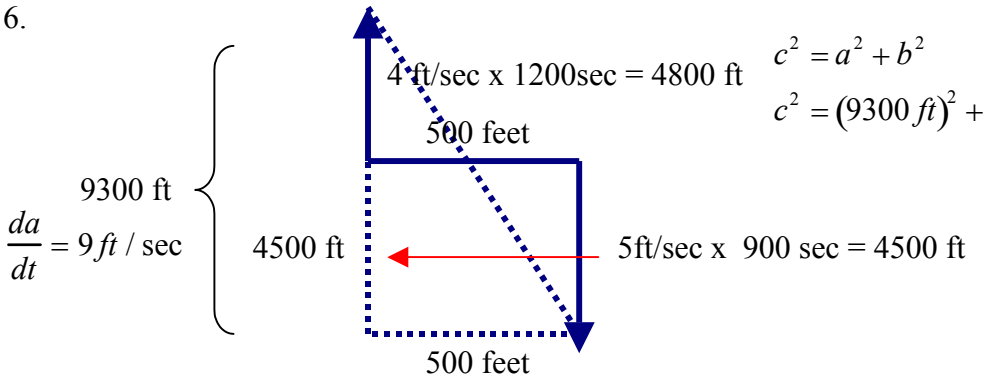
5. $c^2 = a^2 + b^2 \Rightarrow 2c\frac{dc}{dt} = 2a\frac{da}{dt} + 2b\frac{db}{dt} \Rightarrow c\frac{dc}{dt} = a\frac{da}{dt} + b\frac{db}{dt}$

For $a = 10km, b = 100km, c = 100.4mi, \frac{da}{dt} = 35km/hr, \frac{db}{dt} = -25km/hr, \frac{dc}{dt} = ?$

$100.5km \cdot \frac{dc}{dt} = 10km \cdot -35km/hr + 100km \cdot 25km/hr \Rightarrow$

$100.5km \cdot \frac{dc}{dt} = 2150km^2/hr \Rightarrow \frac{dc}{dt} = 21.4km/hr$

6.



$c^2 = a^2 + b^2$

$c^2 = (9300ft)^2 + (500ft)^2 \Rightarrow c = 9313.4ft$

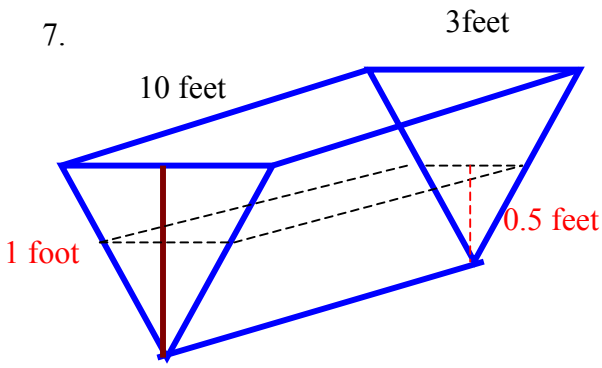
6. $c^2 = a^2 + b^2 \Rightarrow 2c\frac{dc}{dt} = 2a\frac{da}{dt} + 2b\frac{db}{dt} \Rightarrow c\frac{dc}{dt} = a\frac{da}{dt} + b\frac{db}{dt}$

For $a = 9300ft, b = 500ft, c = 9313.4ft, \frac{da}{dt} = 9ft/\sec, \frac{db}{dt} = 0, \frac{dc}{dt} = ?$

$9313.4ft \cdot \frac{dc}{dt} = 9300ft \cdot 9ft/\sec + 500ft \cdot 0 \Rightarrow$

$9313.4ft \cdot \frac{dc}{dt} = 83700ft^2 \sec \Rightarrow \frac{dc}{dt} = 8.98ft/\sec$

7.



$\frac{b}{h} = \frac{3}{1} \Rightarrow b = 3h$ because of similar triangles

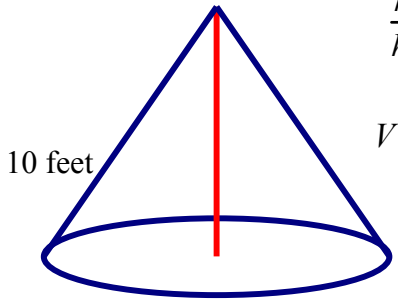
Volume = area of triangle x length

$V = \frac{1}{2}bh \cdot l \Rightarrow V = \frac{1}{2}3h \cdot h \cdot 10 \Rightarrow V = 15h^2$

$V = 15feet \cdot h^2 \Rightarrow \frac{dV}{dt} = 15feet \cdot 2h\frac{dh}{dt} \Rightarrow \frac{dV}{dt} = 30feet \cdot h\frac{dh}{dt}$, For $h = 0.5feet, \frac{dV}{dt} = 12ft^3/\min$

$12ft^3/\min = 30feet(0.5feet)\frac{dh}{dt} \Rightarrow 0.8feet.\min = \frac{dh}{dt}$

8.



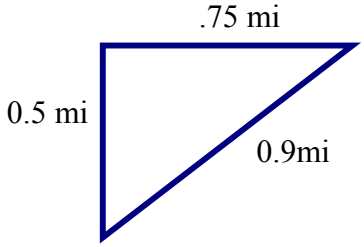
$$\frac{r}{h}=\frac{.5}{1}\Rightarrow r=.5h$$

$$V=\frac{1}{3}\pi r^2h\Rightarrow V=\frac{1}{3}\pi(.5h)^2h\Rightarrow V=\frac{1}{.75}\pi h^3$$

$$\frac{dV}{dt}=\frac{1}{.75}\pi\cdot 3h^2\frac{dh}{dt}\Rightarrow\frac{dV}{dt}=\frac{\pi}{.25}h^2\frac{dh}{dt}\quad\text{For } h=10\,feet,\frac{dV}{dt}=12\,ft^3/\min$$

$$12\,ft^3/\min=\frac{\pi}{.75}(10\,feet)^2\frac{dh}{dt}\Rightarrow\frac{9}{100\pi}\,ft/\min=\frac{dh}{dt}$$

9.



$$c^2=a^2+b^2$$

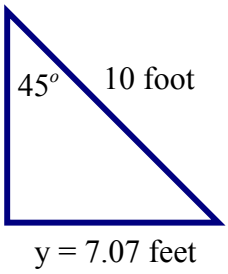
$$c^2=(.75mi)^2+(.5mi)^2\Rightarrow c=0.9\,mi$$

$$c^2=a^2+b^2\Rightarrow 2c\frac{dc}{dt}=2a\frac{da}{dt}+2b\frac{db}{dt}\Rightarrow c\frac{dc}{dt}=a\frac{da}{dt}+b\frac{db}{dt}$$

$$\text{For } a=.75mi,b=.5mi,c=.9mi,\frac{da}{dt}=3mi/hr,\frac{db}{dt}=2mi/hr,\frac{dc}{dt}=?$$

$$.9mi\cdot\frac{dc}{dt}=.75mi\cdot 3mi/hr+.5mi\cdot 2mi/hr\Rightarrow .9mi\cdot\frac{dc}{dt}=2.25mi^2/hr+1mi^2/hr\Rightarrow\frac{dc}{dt}=3.61mi/hr$$

10.



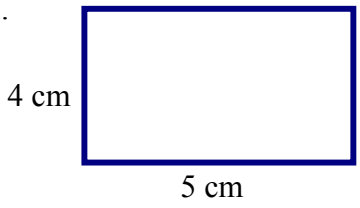
$$\sin\theta=\frac{opp}{hyp}\Rightarrow\sin 45^o=\frac{y}{10\,feet}\Rightarrow y=7.07\,feet$$

$$\sin\theta=\frac{y}{c}\Rightarrow c\sin\theta=y\Rightarrow\frac{dc}{dt}\sin\theta+\cos\theta\cdot\frac{d\theta}{dt}\cdot c=\frac{dy}{dt}$$

$$\text{For } y=7.07\,feet,c=10\,feet,\sin\theta=\sin 45^o=0.707,\cos\theta=\cos 45^o=0.707,\frac{dy}{dt}=2\,ft/\sec,\frac{d\theta}{dt}=?,\frac{dc}{dt}=0$$

$$0\cdot(0.707)+(0.707)\frac{d\theta}{dt}\cdot 10\,feet=2\,ft/\sec\Rightarrow\frac{d\theta}{dt}=0.28^R/\sec=.28*57.3^o=10.04^o/\sec$$

11.

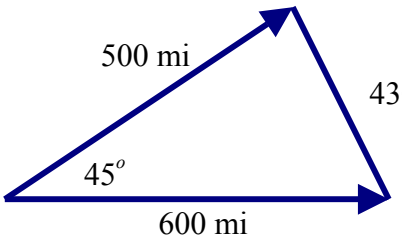


$$A = lw \Rightarrow \frac{dA}{dt} = \frac{dl}{dt} \cdot w + \frac{dw}{dt} \cdot l$$

For $l = 5cm, w = 4cm, \frac{dl}{dt} = 3cm / \sec, \frac{dw}{dt} = 2cm / \sec$

$$\frac{dA}{dt} = 3cm / \sec \cdot 4cm + 2cm / \sec \cdot 5cm \Rightarrow \frac{dA}{dt} = 22cm^2 / \sec$$

12.



$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

$$c^2 = (600mi^2) + (500mi)^2 - 2(600mi)(500mi)\cos 45^o$$

$$c = 430.97mi$$

since the angle does not change we can convert $\cos 45^o$ into 0.707 and the formula becomes $c^2 = a^2 + b^2 - 1.414ab$

$$c^2 = a^2 + b^2 - 1.414ab \Rightarrow 2c\frac{dc}{dt} = 2a\frac{da}{dt} + 2b\frac{db}{dt} + \left(-1.414\frac{da}{dt} \cdot b + \frac{db}{dt} \cdot -1.414a\right)$$

$$For a = 600mi, b = 500mi, c = 430.97mi, \frac{da}{dt} = 400mi / hr, \frac{db}{dt} = 500mi / hr, \frac{dc}{dt} = ?$$

$$2(430.97mi)\frac{dc}{dt} = 2(600mi)(400mi / hr) + 2(500mi)(500mi)$$

$$-1.414(400mi / hr)(500mi) - 1.414(500mi / hr)(600mi)$$

$$861.95mi\frac{dc}{dt} = 27300mi^2 / hr$$

$$\frac{dc}{dt} = 316.72mi / hr$$