Related Rate Problems

1. Find $\dfrac{dy}{dt}$

   a) $y^2 = x^2(x+1), \dfrac{dx}{dt} = a, x = 1, y = -\sqrt{2}$

   b) $y = \dfrac{1}{2}(e^x + e^{-x}), \dfrac{dx}{dt} = 1, x = 1$

2. A point moves along the upper half of the curve $y^2 = 2x+1$ in such a way that $\dfrac{dx}{dt} = \sqrt{2x+1}$. Find $\dfrac{dy}{dt}$ when x = 4.

3. Let $u = (x-1)^3$ and $v = (x+1)^3$, where x is a differentiable function of "t". If $\dfrac{du}{dt} = 6$ when $\dfrac{dx}{dt} = \dfrac{1}{2}$, what is $\dfrac{dv}{dt}$ then?

4. Water is flowing at a rate of 5 cubic feet per minute into a tank in the form of a cone of altitude 20 feet and base radius of 10 feet and with its vertex in the downward direction. Find the rate at which the uncovered surface of the conical tank is decreasing when the water is 8 feet deep.

5. At noon of a certain day, ship A is 60 miles due north of ship B. If ship A sails east at 15 miles per hour and B sails north at 12 miles per hour, determine how rapidly the distance between them is changing 4 hours later. Is it decreasing or increasing?

6. One ship leaves port and steams due north at 10 knots. Three hours later another ship leaves the same port and steams due west at 30 knots per hour. How fast is the distance between them increasing when the first ship has been out of port for 5 hours?

7. The area of an equilateral triangle is increasing at a rate of 4 square inches per minute. At what rate is one side increasing when the area is 10 square inches?

8. A small spherical balloon is being inflated at a rate of 1 cubic foot per minute. At what rate is the diameter increasing2 seconds after inflation begins? How fast is the surface area increasing?

9. At a given instant the legs of a right triangle are 5 cm. and 12 cm. long. If the short leg is increasing at a rate of 1 cm per second and the long leg is decreasing at 2 cm per second, how fast is the hypotenuse changing?

10. A ladder 25 feet long is leaning against a wall. If the lower end of the ladder is pulled away from the wall at a rate of 1 foot per second, what rate is the other end slipping down the wall when it is 24 feet from the ground?

11. The dimensions of a box are 10 inches, 12 inches and 16 inches. If the shorter sides are decreasing at a rate of 0.2 inches per minute and the longest side is increasing at a rate of 0.3 inches per minute, how fast is the volume changing?

12. A baseball diamond is a 90 foot square. A ball is batted along the third-base line at a constant speed of 100 feet per second. How fast is its distance from first base changing when a) it is halfway to third base, b) it reaches third base?

13. A pile of sand being dumped forms a right circular cone in which the altitude is 2/3 the diameter. If the sand is dumped at 3 cubic feet per second, find the rate of increase of the diameter of the pile when it is 6 feet high.

14. A cylindrical reservoir is 10 feet high and 10 feet in diameter. Water flows in at a rate of 1 ½ cubic feet per second.  How fast is the water rising?
15. A vat is in the shape of a rectangular parallelpiped, with a horizontal base having dimensions 4 feet by 6 feet.  Oil is flowing into the vat at the rate of 30 cubic feet per minute. How fast is the oil level raising?
16. A trough that is 12 feet long and 2 feet high is 2 feet wide at the top and has triangular ends.  If water is put into the tank at the rate of  1 cubic foot per minute, how fast is the depth increasing when it is 1.5 feet deep.
17. A hemispherical bowl is 2 feet in diameter. Liquid is poured in at a rate of 200 cubic inches per minute.  How fast is the surface rising as it overflows?
18. A long level railway bridge passes over a railroad track which is 100 feet below it and at right angles to it.  If an automobile travelling 45 miles per hour is directly above a train going 60 miles per hour, how fast will they be separating 10 seconds later?
19. A pebble thrown into a still pond produces concentric circular ripples. If the radius of the largest ripple increases at 2 feet per second, how fast is the area of disturbance within the ripple increasing when the radius is 15 feet?
20. A cylindrical steel ingot is shrinking in cooling with a base radius decreasing at 0.001 inches per minute and the latitude decreasing at 0.005 inches per minute.  At an instant when the radius is 20inches and the altitude is 100 inches , find the rate of decrease in the volume of the ingot.
21. Each edge of a cube is expanding at the rate of 1 cm per second. How fast is the volume changing when the length of each edge is (a0 5 cm? (b) 10cm? (c) x cm?
22. A cone has a base of radius "r" and altitude "h". If "r" is increasing at the rate of 2/3 inches per second and "h" is decreasing at a rate of –1 inch per second, at what rate is the volume of the cone changing when r = 4 and h = 4?
23. A man 6 feet tall is walking horizontally at the rate of 84 feet per minute directly toward a light which is 20 feet above the ground. At what rate is the length of his shadow changing?
24. A lamp on a post is 24 feet high.  A man of height 5 feet 6 inches is walking directly away from the lamp post at the rate of 4 feet per second.  How fast is his shadow lengthening when he is 21 feet from the post?