**HIGHER ORDER DERIVATIVES**

1. Find the first and second order derivatives of the given function.

a)
$$f(x) = x^4 - 2x^3 + 4x^2 - 6$$
$$f'(x) = 4x^3 - 6x^2 + 8x$$
$$f''(x) = 12x^2 - 12x + 8$$

b)
$$f(x) = x^{10} + 4x^7 - 2x^3 + 2x$$
$$f'(x) = 10x^9 + 28x^6 - 6x^2 + 2$$
$$f''(x) = 90x^8 + 168x^5 - 12x$$

c)
$$f(x) = \sqrt{x^2+1} = \left(x^2+1\right)^{1/2}$$
$$f'(x) = \tfrac{1}{2}\left(x^2+1\right)^{-1/2} \cdot 2x = x\left(x^2+1\right)^{-1/2}$$
$$f''(x) = 1 \cdot \left(x^2+1\right)^{-1/2} + \left(-\tfrac{1}{2}\right)\left(x^2+1\right)^{-3/2} \cdot 2x \cdot x$$
$$= \left(x^2+1\right)^{-3/2}\left[\left(x^2+1\right) - x^2\right] = \frac{1}{\left(x^2+1\right)^{3/2}}$$

d)
$$f(x) = \sqrt[3]{x} + \sqrt{x} = x^{1/3} + x^{1/2}$$
$$f'(x) = \tfrac{1}{3}x^{-2/3} + \tfrac{1}{2}x^{-1/2}$$
$$f''(x) = \tfrac{1}{3} \cdot -\tfrac{2}{3}x^{-5/3} + \tfrac{1}{2} \cdot -\tfrac{1}{2}x^{-3/2} = \tfrac{1}{3} \cdot -\tfrac{2}{3}x^{-10/6} + \tfrac{1}{2} \cdot -\tfrac{1}{2}x^{-9/6}$$
$$= x^{-10/6}\left(-\tfrac{2}{9} - \tfrac{1}{4}x^{1/6}\right) = \frac{\left(-\tfrac{2}{9} - \tfrac{1}{4}x^{1/6}\right)}{x^{10/6}} = \frac{-\left(8 + 9x^{1/6}\right)}{36x^{10/6}}$$

e)
$$f(x) = (3x+2)^3$$
$$f'(x) = 3(3x+2)^2 \cdot 3 = 9(3x+2)^2$$
$$f''(x) = 9 \cdot 2(3x+2)^1 \cdot 3 = 54(3x+2)$$

f)
$$f(x) = \ln(3x+4)^3$$
$$f'(x) = \frac{1}{(3x+4)^3} \cdot 3(3x+4)^2 \cdot 3 = 9(3x+4)^{-1}$$
$$f''(x) = 9 \cdot -1(3x+4)^{-2} \cdot 3 = \frac{-27}{(3x+4)^2}$$

g)
$$f(x) = e^{(5x+3)}$$
$$f'(x) = e^{(5x+3)} \cdot 5 = 5e^{(5x+3)}$$
$$f''(x) = 5 \cdot e^{(5x+3)} \cdot 5 = 25e^{(5x+3)}$$

h)
$$f(x) = \left(4x+1\right)^{3/4}$$
$$f'(x) = \tfrac{3}{4}\left(4x+1\right)^{-1/4} \cdot 4 = 3\left(4x+1\right)^{-1/4}$$
$$f''(x) = 3 \cdot -\tfrac{1}{4}\left(4x+1\right)^{-5/4} \cdot 4 = -3\left(4x+1\right)^{-5/4}$$
$$= \frac{-3}{\left(4x+1\right)^{5/4}}$$

2. Find the third derivative of the given function.

$$f(x)=\sqrt{5x-1}=\left(5x-1\right)^{1/2}$$

$$f'(x)=\tfrac{1}{2}\left(5x-1\right)^{-1/2}\cdot 5=\tfrac{5}{2}\left(5x-1\right)^{-1/2}$$

a) $$f''(x)=\tfrac{5}{2}\cdot -\tfrac{1}{2}\left(5x-1\right)^{-3/2}\cdot 5=-\tfrac{25}{4}\left(5x-1\right)^{-3/2}$$

$$f'''(x)=-\tfrac{25}{4}\cdot -\tfrac{3}{2}\left(5x-1\right)^{-5/2}\cdot 5=\tfrac{375}{8}\left(5x-1\right)^{-5/2}$$

$$=\frac{375}{8\left(5x-1\right)^{5/2}}$$

$$f(x)=\frac{1-x}{1+x}=\left(1-x\right)\left(1+x\right)^{-1}$$

$$f'(x)=-1\left(1+x\right)^{-1}+-1\cdot\left(1+x\right)^{-2}\cdot 1\cdot\left(1-x\right)=\left(1+x\right)^{-2}\left[-1(1+x)-1(1-x)\right]=-2\left(1+x\right)^{-2}$$

b) $$f''(x)=-2\cdot -2\left(1+x\right)^{-3}\cdot 1=4\left(1+x\right)^{-3}$$

$$F'''(X)=4\cdot -3\left(1+x\right)^{-4}\cdot 1=-12\left(1+x\right)^{-4}=\frac{-12}{\left(1+x\right)^{4}}$$

$$f(x)=\frac{5}{(1+x^2)}=5(1+x^2)^{-1}$$

$$f'(x)=5\cdot -1(1+x^2)^{-2}\cdot 2x=-10x(1+x^2)^{-2}$$

$$f''(x)=-10(1+x^2)^{-2}+-2(1+x^2)^{-3}\cdot 2x\cdot -10x=-10(1+x^2)^{-2}+40x^2(1+x^2)^{-3}$$

$$=10(1+x^2)^{-3}\left[-(1+x^2)+4x^2\right]=10(1+x^2)^{-3}\left(3x^2-1\right)$$

c) $$f'''(x)=10\cdot -3(1+x^2)^{-4}\cdot 2x\cdot\left(3x^2-1\right)+6x\cdot 10(1+x^2)^{-3}$$

$$=-60x(1+x^2)^{-4}\left(3x^2-1\right)+60x(1+x^2)^{-3}$$

$$=60x(1+x^2)^{-4}\left[-\left(3x^2-1\right)+1(1+x^2)\right]$$

$$=60x(1+x^2)^{-4}(-2x^2+2)=\frac{120(1-x^2)}{(1+x^2)^4}$$

3. If $f(x) = (2-3x)^{(-1/2)}$, find $f(0)$, $f'(0)$, $f''(0)$, and $f'''(0)$

$$f(x) = \left(2-3x\right)^{-1/2} \Rightarrow$$

$$f(0) = \left(2-3(0)\right)^{-1/2} = 2^{-1/2} = \frac{1}{2^{1/2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$f'(x) = -1/2\left(2-3x\right)^{-3/2} \cdot -3 = 3/2\left(2-3x\right)^{-3/2} = \frac{3}{2\left(2-3x\right)^{3/2}} \Rightarrow$$

$$f'(x) = \frac{3}{2\left(2-3(0)\right)^{3/2}} = \frac{3}{2\sqrt{2^3}} = \frac{3\sqrt{2}}{8}$$

$$f''(x) = 3/2 \cdot -3/2\left(2-3x\right)^{-5/2} \cdot -3 = 27/4\left(2-3x\right)^{-5/2} = \frac{27}{4\left(2-3x\right)^{5/2}} \Rightarrow$$

$$f''(x) = \frac{27}{4\left(2-3(0)\right)^{5/2}} = \frac{27}{4\sqrt{2^5}} = \frac{27\sqrt{2}}{32}$$

$$f'''(x) = 27/4 \cdot -5/2\left(2-3x\right)^{-7/2} \cdot -3 = 405/8\left(2-3x\right)^{-7/2} = \frac{405}{8\left(2-3x\right)^{7/2}} \Rightarrow$$

$$f'''(x) = \frac{405}{8\left(2-3(0)\right)^{7/2}} = \frac{405}{8\sqrt{2^7}} = \frac{405\sqrt{2}}{128}$$

4. If $f(x) = (2 - t^2)^6$, find $f(0)$, $f'(0)$, $f''(0)$, and $f'''(0)$

$$f(t) = (2-t^2)^6 \Rightarrow$$

$$f(0) = (2-(0)^2)^6 = 2^6 = 64$$

$$f'(t) = 6(2-t^2)^5 \cdot -2t = -12t(2-t^2)^5 \Rightarrow$$

$$f'(0) = -12(0)(2-(0)^2)^5 = 0$$

$$f'(t) = -12(2-t^2)^5 + 5(2-t^2)^4 \cdot -2t \cdot -12t = -12(2-t^2)^4\left[(2-t^2)+5(-2t)(t)\right] =$$

$$-12(2-t^2)^4(2-11t^2) \Rightarrow$$

$$f''(0) = -12(2-(0)^2)^4(2-11(0)^2) = -12(2)^4(2) = -384$$

$$f'''(t) = -12 \cdot 4(2-t^2)^3 \cdot -2t \cdot (2-11t^2) + -22t \cdot -12(2-t^2)^4 =$$

$$24t(2-t^2)^3\left[4(2-11t^2)+11\right] = 24t(2-t^2)^3(19-44t^2) \Rightarrow$$

$$f'''(0) = 24(0)\left[2-(0)^2\right]^3\left[19-44(0)^2\right] = 0$$

5. Find a second-degree polynomial "$f$" such that $f(2) = 5, f'(2) = 3$, and $f''(2) = 2$

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$$f'(x) = 2x + c \Rightarrow 3 = 2(2) + c \Rightarrow 3 = 4 + c \Rightarrow -1 = c \Rightarrow$$
$$f'(x) = 2x - 1$$
$$f(x) = x^2 - x + c \Rightarrow 5 = (2)^2 - (2) + c \Rightarrow 5 = 4 - 2 + c \Rightarrow 3 = c \Rightarrow$$
$$f(x) = x^2 - x + 3$$

6. Find a third-degree polynomial "$f$" such that $f(1) = 1, f'(1) = 3, f''(1) = 6$, and $f'''(1) = 12$.

$f(1) = 1, f'(1) = 3, f''(1) = 6$, and $f'''(1) = 12$

$$f''(x) = 12x + c \Rightarrow 6 = 12(1) + c \Rightarrow 6 = 12 + c \Rightarrow -6 = c \Rightarrow$$
$$f''(x) = 12x - 6$$
$$f'(x) = 6x^2 - 6x + c \Rightarrow 3 = 6(1)^2 - 6(1) + c \Rightarrow 3 = 6 - 6 + c \Rightarrow 3 = c \Rightarrow$$
$$f'(x) = 6x^2 - 6x + 3$$
$$f(x) = 2x^3 - 3x^2 + 3x + c \Rightarrow 1 = 2(1)^3 - 3(1)^2 + 3(1) + c \Rightarrow 1 = 2 - 3 + 3 + c \Rightarrow -1 = c \Rightarrow$$
$$f(x) = 2x^3 - 3x^2 + 3x - 1$$

7. Note: the first derivative represents the velocity of an object as a function of time.
: the second derivative represents the instantaneous rate of change of velocity with respect to time (acceleration)

Each equation represents the motion of a given particle with distance in meters and time in seconds. Find (a) the velocity and acceleration as a function of time, b) the acceleration after 1 sec, and c) the acceleration at the instants when the velocity is 0.

a)

$a)\ s(t) = t^3 - 3t$

$s'(t) = 3t^2 - 3$

$s''(t) = 6t$

$b)\ s''(t) = 6(1) = 6$

$c)\ s'(t) = 3t^2 - 3 \Rightarrow 0 = 3t^2 - 3 \Rightarrow 3 = 3t^2 \Rightarrow 1 = t^2 \Rightarrow t = 1 \therefore s''(t) = 6(1) = 6$

b)

$a)\ s(t) = t^2 - t + 1$

$s'(t) = 2t - 1$

$s''(t) = 2$

$b)\ s''(t) = 2$

$c)\ s'(t) = 2t - 1 \Rightarrow 0 = 2t - 1 \Rightarrow 1 = 2t \Rightarrow t = \frac{1}{2} \therefore s''(t) = 2$

$a)\ s(t) = 2t^3 - 7t^2 + 4t + 1$

$s'(t) = 6t^2 - 14t + 4$

$s''(t) = 12t - 14$

$b)\ s''(t) = 12t - 14 \Rightarrow s''(1) = 12(1) - 14 \Rightarrow s''(1) = -2$

c) $c)\ s'(t) = 6t^2 - 14t + 4 \Rightarrow 0 = 6t^2 - 14t + 4 \Rightarrow 0 = 3t^2 - 7t + 2 \Rightarrow$

$0 = (3t - 1)(t - 2) \Rightarrow t = {}^1\!/_3 \text{ or } 2 \therefore$

$s''\left({}^1\!/_3\right) = 12\left({}^1\!/_3\right) - 14 \Rightarrow s''\left({}^1\!/_3\right) = 4 - 14 \Rightarrow s''\left({}^1\!/_3\right) = -10$

$s''(2) = 12(2) - 14 \Rightarrow s''(2) = 24 - 14 \Rightarrow s''(2) = 10$