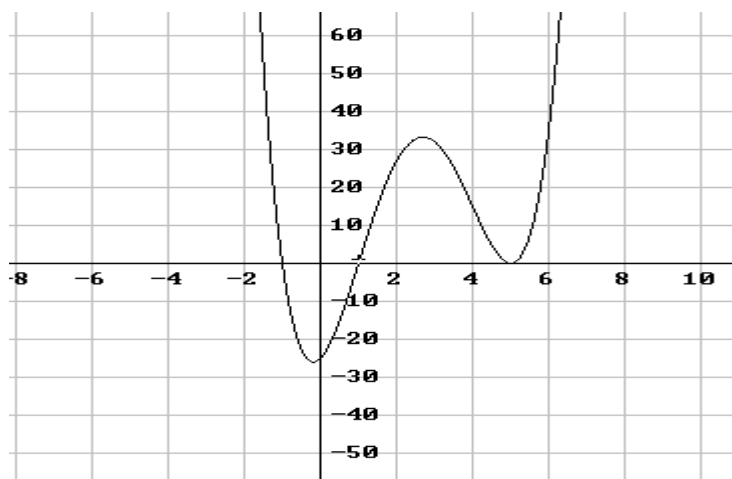
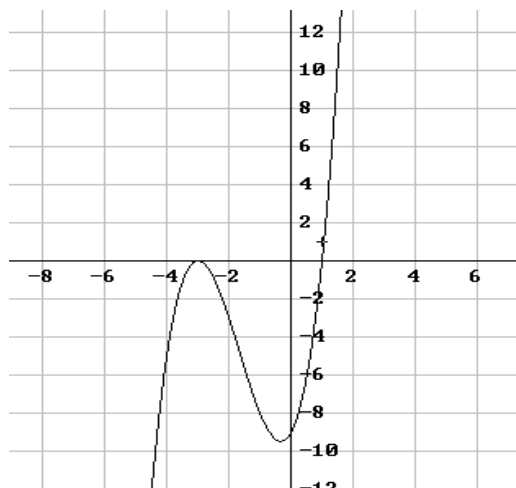


Polynomial Function Exam

Equation	$x^3 + 5x^2 + 3x - 9$	$x^4 - 10x^3 + 24x^2 + 10x - 25$		$x^3 + 5x^2 + 3x - 9$	$x^4 - 10x^3 + 24x^2 + 10x - 25$
1. degree of equation	3	4	7. possible number of positive real roots	1	2
2. the value of the constant	-9	-25	8. possible number of negative real roots	1	1
3. the value of the leading coefficient	1	1	9. possible number of imaginary roots	0	0
4. the value of the y intercept	-9	-25	10. write out the possible factors	$(x + 3)^2(x - 1)$	$(x - 1)(x + 1)(x - 5)^2$
5. where the graphs starts	Lower left	Upper left	11. identify critical zeros (values, x-intercepts)	-3, 1	-1, 1, 5
6. where the graph finishes	Upper right	Upper Right	12. multiplicity of each factor	$2 - (x + 3),$ $1 - (x - 1)$	$2 - (x - 5), 1 - (x - 1),$ $1 - (x + 1)$

Sketch each graph



Sketch the graph of each of the two given equations

1. Determine the roots of each quadratic equation by the indicated method.

a) Factoring

$$2x^2 + 8x - 10 = 0$$



$$\begin{aligned} 2(x^2 + 4x - 5) &= 0 \\ 2(x+5)(x-1) &= 0 \\ x+5=0 \text{ or } x-1=0 \\ x &= -5 \text{ or } x=1 \\ &\{-5, 1\} \end{aligned}$$

b) Complete the Trinomial Square

$$2x^2 + 3x - 1 = 0$$

$$\begin{aligned} x^2 + \frac{3}{2}x &= \frac{1}{2} \\ x^2 + \frac{3}{2}x + \frac{9}{16} &= \frac{1}{2} + \frac{9}{16} = \frac{1 \cdot 8 + 9}{16} \\ \left(x + \frac{3}{4}\right)^2 &= \frac{17}{16} \\ \sqrt{\left(x + \frac{3}{4}\right)^2} &= \pm \sqrt{\frac{17}{16}} \\ x + \frac{3}{4} &= \frac{\pm \sqrt{17}}{4} \\ x &= -\frac{3}{4} \pm \frac{\sqrt{17}}{4} \Rightarrow \left\{ \frac{-3 \pm \sqrt{17}}{4} \right\} \end{aligned}$$

c) Quadratic Formula

$$-4x^2 + 5x + 3 = 0$$



$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-(5) \pm \sqrt{(5)^2 - 4(-4)(3)}}{2(-4)} \\ x &= \frac{-5 \pm \sqrt{25 + 48}}{-8} \\ x &= \frac{-5 \pm \sqrt{73}}{-8} \Rightarrow \left\{ \frac{-5 \pm \sqrt{73}}{-8} \right\} \end{aligned}$$

2. Determine the sum and the product of the roots for the following equation:

$$4x^2 - 5x + 6 = 0$$

$$\text{sum} = -\frac{b}{a} = -\frac{-5}{4} = \frac{5}{4}, \quad \text{product} = \frac{c}{a} = \frac{6}{4} = \frac{3}{2}$$

3. Determine the equation of quadratic function if the roots are $(2 \pm \sqrt{3})$

$$\begin{aligned} x^2 - (r_1 + r_2)x + r_1r_2 &= 0 \\ x^2 - (2 + \sqrt{3} + 2 - \sqrt{3}) + (2 + \sqrt{3})(2 - \sqrt{3}) &= 0 \\ x^2 - (4)x + (4 - 3) &= 0 \\ x^2 - 4x + 1 &= 0 \end{aligned}$$

4. Determine the value of the discriminant and the nature of the roots of the quadratic function

$$5x^2 - 3x - 2 = 0.$$

$$b^2 - 4ac$$

$$(-3)^2 - 4(5)(-2) \longrightarrow \text{nature of roots} = 2 \text{ real rational roots}$$

$$9 + 40$$

$$49$$

5. Determine the roots of one of the following polynomial functions (Do one of the following)

a) $3x^4 - 8x^2 - 3 = 0$

b) $4x - 4\sqrt{x} - 15 = 0$

Let $z = x^2$ by factoring

Let $z = \sqrt{x}$ by factoring

$$3z^2 - 8z - 3 = 0$$

$$4z^2 - 4z - 15 = 0$$

$$(3z+1)(z-3) = 0$$

$$(2z-5)(2z+3) = 0$$

$$3z+1 = 0 \text{ or } z-3 = 0$$

$$2z-5 = 0 \text{ or } 2z+3 = 0$$

$$z = -\frac{1}{3} \text{ or } z = 3$$

$$z = \frac{5}{2} \text{ or } z = -\frac{3}{2}$$

$$\therefore x^2 = -\frac{1}{3} \text{ or } x^2 = 3$$

$$\therefore \sqrt{x} = \frac{5}{2} \text{ or } \sqrt{x} = -\frac{3}{2}$$

$$x = \{\} \text{ or } x = \pm\sqrt{3}$$

$$x = \frac{25}{4} \text{ or } x = \frac{9}{4}$$

$$\text{Solution Set} = \{\pm\sqrt{3}\}$$

$$\text{Solution Set} = \left\{\frac{25}{4}, \frac{9}{4}\right\}$$

Let $z = x^2$ by quadratic formula

Let $z = \sqrt{x}$ by quadratic formula

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$z = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(-3)}}{2(3)}$$

$$z = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(-15)}}{2(4)}$$

$$z = \frac{8 \pm \sqrt{64 + 36}}{6} = \frac{8 \pm \sqrt{100}}{6} = \frac{8 \pm 10}{6}$$

$$z = \frac{4 \pm \sqrt{16 + 240}}{8} = \frac{4 \pm \sqrt{256}}{8} = \frac{4 \pm 16}{8}$$

$$z = \frac{8+10}{6} = \frac{18}{6} = 3, z = \frac{8-10}{6} = \frac{-2}{6} = -\frac{1}{3}$$

$$z = \frac{4+16}{8} = \frac{20}{8} = \frac{5}{2} \text{ and } z = \frac{4-16}{8} = \frac{-12}{8} = -\frac{3}{2}$$

see above for rest of solution

see above for rest of solution

6. State the inverse for each of the following:

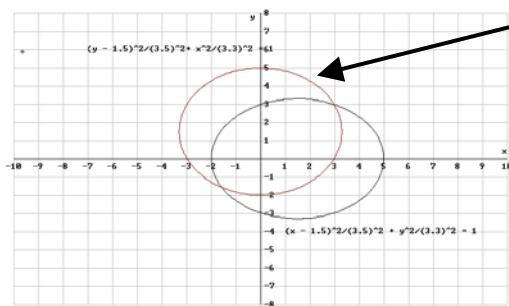
a) $5x - 2y = 11$

Inverse $5x - 2y = 11$

b) $\begin{array}{c|ccc} x & -3 & 1 & 5 \\ y & 1 & 2 & -4 \end{array}$

Inverse $\begin{array}{c|ccc} x & 1 & 2 & -4 \\ y & -3 & 1 & 5 \end{array}$

c)



7. State the reciprocal for each of the following:

a) $y = 3x - 5$

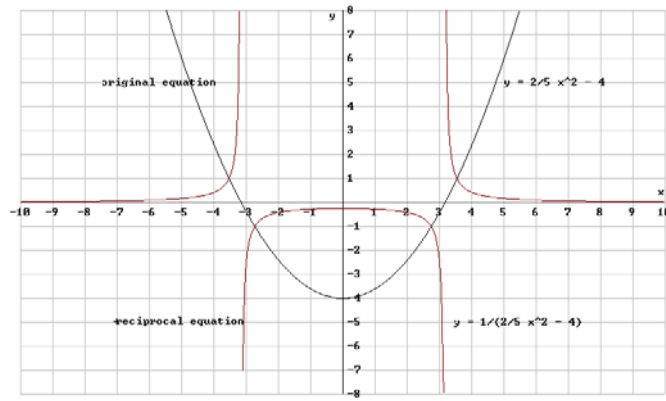
reciprocal $y = \frac{1}{3x - 5}$

b)

x	2	4	5
y	0	2	7

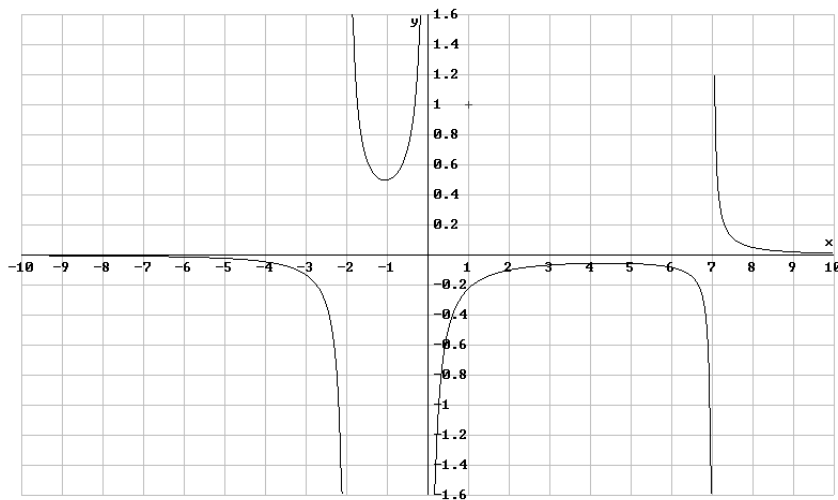
reciprocal x	-2	4	5
y	1/0	1/2	1/7

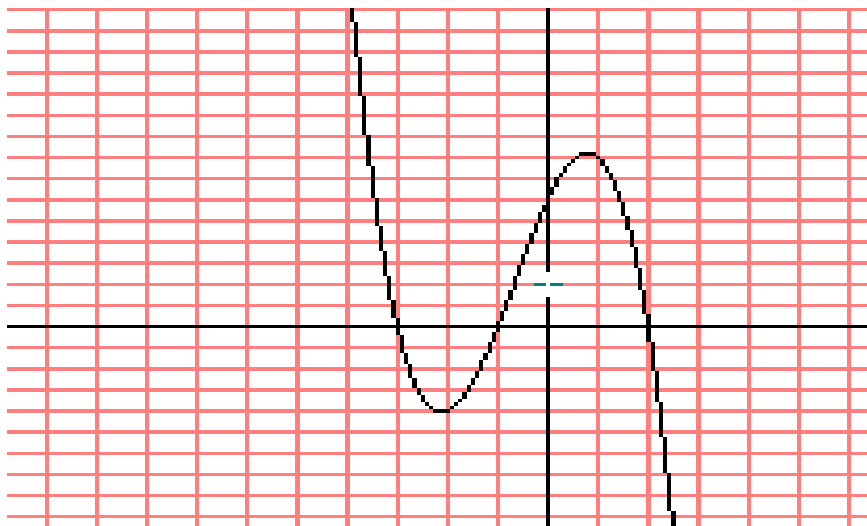
↑
undefined



c)

8. Graph the following rational expression: $y = \frac{4}{x^3 - 5x^2 - 14x}$





Complete the table using the above graphs

1. possible degree of the function	3	5	7.	1	1
			number		
			of peaks		
2. value of the y-intercept	6	-2	8.number of valleys	1	1
3. number of positive real roots	1	1	9. critical zeros	-3, -1, 1	-2, -1, 1
4. number of negative real roots	2	2	10. the factors containing critical	$(x + 3)(x + 1)(x - 1)$	$(x + 2)(x + 1)$
			zeros		$(x - 1)$
5. number of imaginary roots	0	0	11. the equation	$y = (x + 3)(x + 1)(x - 1)$	$y = (x + 2)(x + 1)^3(x - 1)$
6. number of times graph changes direction	2	2	12. multiplicity of each factor	1	$3 - (x + 1) 1$
					$- (x + 2), 1 -$
					$(x - 1)$